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Review

A Review of two-phase flow dynamic instabilities in tube boiling systems

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Abstract

The earliest research in the field of two-phase flow was conducted by Lorentz (1909). The studies on the analysis of two-phase flow instabilities by Ledinegg (1938) created considerable interest concerning the phenomenon of thermally induced flow instability in two-phase flow systems. The objective of this review is to sum up the experimental and theoretical work carried out by various investigators over a period of several years, demonstrating and explaining three main instability modes of two-phase flow dynamic instabilities, namely, density-wave type, pressure-drop type and thermal oscillations, encountered in various boiling flow channel systems. The typical experimental investigations of these instabilities in tube boiling systems are indicated and the most popular models to predict the two-phase flow dynamic instabilities, namely the homogenous flow model and the drift-flux models are clarified with the solution examples and the validation of the model results with experimental findings are also provided.

Keywords: Two-phase flow instabilities; Pressure-drop; Density-wave; Thermal oscillations

Contents

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Nomenclature

A	heater inner surface area, m^2
A	speed of sound, m/s
c_h	specific heat of heater wall material, kJ/(kg K)
C	wetted perimeter of the heater tube, m
c_p	specific heat capacity at constant pressure, kJ/(kg K)
D	inner diameter of the heater tube, m
D_e	equivalent diameter, m
f	friction factor, dimensionless
F_m	two-phase flow friction multiplier, dimensionless
g	gravitational acceleration, 9.806 m/s^2
G	fluid mass velocity $= \rho u$, kg/(m^2 s)
G_1	inlet fluid mass velocity into surge tank, kg/(m^2 s)
G_2	outlet fluid mass velocity from surge tank, kg/(m^2 s)
h	specific enthalpy of the fluid, J/kg
h_s	saturation enthalpy, J/kg
h_e	equivalent enthalpy of the fluid (drift-flux model), J/kg
h_{lv}	latent heat of evaporation, J/kg
k	thermal conductivity of the fluid, W/(m K)
L	tube length, m
L_h	heated length of the tube, m
$\dot{m}$	mass flow rate, kg/s
Nu	Nusselt number $= \alpha d/k$, dimensionless
P	pressure, Pa
P_e	exit pressure, Pa
P_s	surge tank pressure, Pa
P_{sao}	steady-state pressure of air in the surge tank pressure, Pa
P_{sa}	unsteady state pressure of air in the surge tank pressure, Pa
P_o	main tank pressure, Pa
Pr	Prandtl number $= c_p\mu/k$, dimensionless
Q_I	heat input into the fluid, W
Q_o	electrical heat generation rate in the heater wall, W
R	resistance coefficient for inlet restriction, dimensionless
Re	Reynold's number $= \rho ud/\mu$, dimensionless
T	temperature, K
t	time, s
Δt	time increment in the numerical scheme, s
T_i	fluid inlet temperature, °C
u	fluid velocity, m/s
V	volume, m^3
V_{go}	steady-state volume of the gas in the surge tank, m^3
x	quality of the liquid–vapor mixture, dimensionless
z	axial distance along the flow path, m

Greek symbols

α	heat transfer coefficient, W/(m^2 K)
μ	dynamic viscosity of the fluid, Pa s
ρ	density, kg/m^3
Φ	heat input to fluid per unit inner volume of heater, W/m^3
ψ	void fraction, dimensionless
τ_w	shear stress at the wall, N/m^2

Subscripts

e	exit condition
f	fluid parameter
g	gas
h	heater
i	axial label of a node
i	inlet condition
l	liquid
o	steady-state condition
e	exit condition
f	fluid parameter
g	gas
s	surge tank
v	vapor
w	wall condition

1. Introduction

The phenomenon of thermally induced two-phase flow instability is of interest in the design and operation of many industrial systems and equipment, such as steam generators, boiling water reactors, thermosiphons, reboilers, refrigeration plants, and some chemical processing systems.

Oscillations of flow rate and system pressure are undesirable, as they can cause mechanical vibrations, problems of system control, and in extreme circumstances, disturb the heat transfer characteristics so that the heat transfer surface may burn-out. In a recirculating plant, where burn-out must be avoided, flow oscillations could lead to transient burn-out. Under certain circumstances, large flow oscillations can lead to tube failures due to increases in wall temperatures. Another cause of failure would be due to thermal fatigue resulting from continual cycling of the wall temperature; the thermal stresses set up in the wall and the cladding material in nuclear reactor fuel elements can cause mechanical breakdown, leading to more serious accidents, such as release of radioactive materials. It is clear from these examples that the flow instabilities must be avoided, and every effort needs to be made to ensure that any two-phase system has an adequate margin against them.

Flow stability is of particular importance in water-cooled and water-moderated nuclear reactors, and steam generators. The safe operating regime of a two-phase heat exchanger can be determined by the instability threshold values of such system parameters as flow rate, pressure, wall temperatures, and exit mixture quality. The designer's job is to predict the threshold of flow instability so that one can design around it in order to avoid the unwanted instabilities.

2. Classifications of two-phase flow instabilities

A flow is stable if, when disturbed, its new operating conditions tend asymptotically towards the original ones. Mathematically, it is to say that the original operating point is a solution to a system with the property that slight perturbations damp out to produce the original state. Formal discussions of stability can be found in Minorsky [1]. A flow is said to be subject to a static instability if, when disturbed, its new operating conditions tend asymptotically toward the ones that are different from the original ones. In the language of dynamics, it is to say that the original operating point is not a stable equilibrium point, and the system moves to a different equilibrium point which is a stable one.

A flow is said to be subject to a dynamic instability when there is sufficient interaction and delayed feedback between the inertia of flow and compressibility of the two-phase mixture or it may result from multiple feedbacks between flow rate, pressure-drop and the change in density as a result of the rate of vapor generation in a boiling channel. Dynamic instabilities can be characterized as:

1. Density-wave type oscillations.
2. Pressure-drop type oscillations.
3. Acoustic oscillations.
4. Thermal oscillations.

The mechanisms for the dynamic instabilities can be explained by propagation time lags and feedback phenomena present in any two-phase flow system. The momentary disturbance takes some time, which is proportional to the propagation wave speed, in reaching other points along the system. These delayed disturbances along the system are then reflected back to the initial point of the disturbance creating a new disturbance, and so on. Boure et al. [2] proposed the widely accepted classification of two-phase flow instabilities based on the distinction between the static and dynamic instabilities explained above. Lahey [3] and Bergels [4,5] gave more extensive classifications based on Boure's results and included more types of instabilities.

2.1. Mechanism of Ledinegg instability

The earliest research in the field of two-phase flow is ascribed to a German engineer, Lorentz, who investigated

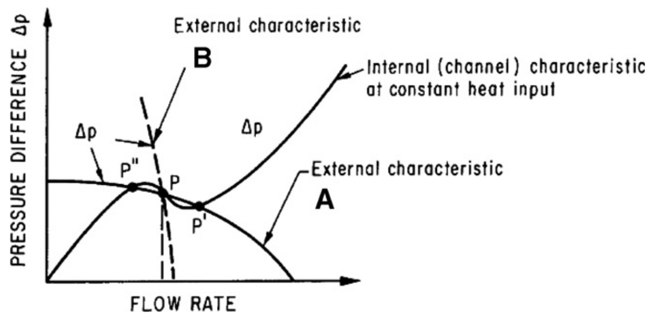

Fig. 1. Ledinegg instability.

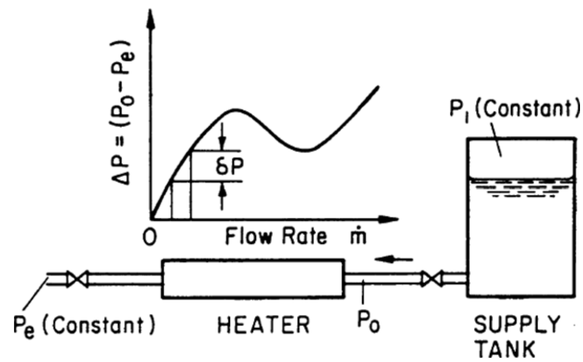

Fig. 2. Simplified system for density-wave type oscillations.

the hydrodynamics of two-phase flows in pipes in 1909 by treating the gas–liquid mixture as a single homogenous fluid [6]. Ledinegg instability involves a sudden change in the flow rate to a lower value [7]. It occurs when the slope of the channel demand pressure-drop versus flow rate curve (internal characteristic curve) is negative and steeper than the loop supply pressure-drop versus flow rate curve (external characteristic curve A) and multiple intersections of the internal and external characteristics. Ledinegg observed that the internal characteristic curve had a negative slope region in addition to two positive slope regions, thus making the flow rate a multi-valued function of the pressure-drop. The scenario of Ledinegg instability is shown in Fig. 1. With an external characteristic (A) less steeper than the internal characteristics, the operation at point P is impossible. A small disturbance to a lower flow would lead to a condition where more pressure-drop was required to sustain the flow than was available from the external system, and the flow rate would decrease further. Therefore, a slight decrease in flow rate will cause a spontaneous shift to pointP''. The new equilibrium point usually corresponds to such a low flow rate that burn-out occurs. It is apparent that it would not be possible to operate to the left of the minimum point on the internal characteristic curve with a flat external characteristic curve. The standard way to avoid the Ledinegg instability is to make the slope of the external characteristic curve steeper than that of the internal.

2.2. Mechanism of density-wave oscillations

An important work towards understanding the mechanism of density-wave oscillations is by Stenning [8] and Stenning et al. [9,10], who started the terminology which was later widely accepted. In the following, we give a simple explanation of the mechanism of density-wave oscillations. A simplified explanation was given by Svanholm [11] for a natural circulation boiling loop. Fig. 2 shows a simple model for density-wave oscillations, which has an evaporator followed by a duct and a flow restriction. With reference to Fig. 2, the mechanism of density-wave oscillations can be explained. The pressure at the inlet and exit reservoirs are kept constant at all times. It is assumed that the rate of vapor generation in the test section is constant at all times. Suppose, that at time $t = 0$, the exit restriction pressure-drop undergoes a sudden infinitesimal drop from its steady-state value. This will cause a drop in the inlet pressure P_o almost instantaneously (at the speed of sound in the fluid) since the exit pressure P_e is constant. The inlet velocity (u_i) now increases infinitesimally since

$$u_i \propto \sqrt{P_I - P_O} \tag{1}$$

This sends a higher density fluid into the test section at $t = 0$. An increased inlet velocity will cause the system pressure-drop to go up (see curve). After a time t, which is the time taken by a particle to reach the exit restriction, an increased pressure-drop causes the inlet pressure P_o to increase since the exit pressure is constant. An increased inlet pressure causes a decrease in inlet velocity (note above equation). A decreased inlet velocity in turn causes the residence time of the particle to go up and thus the fluid has greater enthalpy (and lesser density) when it reaches the restriction. A lower inlet velocity causes a lesser pressure-drop at the restriction' and this starts the cycle again. From this analysis it is evident that it takes around twice the residence time of a fluid particle for a complete set of events to repeat. Typical recordings of the density-wave type oscillations are given in Fig. 10.

2.3. Mechanism of pressure-drop oscillations

With reference to Fig. 3, the following steady-state relations are considered:

$$(P_1 - P_2) = K_1 Q_1^2 \tag{2}$$

$$(P_2 - P_e) = \Psi(Q_2) \tag{3}$$

Here P_1 is the main tank pressure, P_2 the surge tank pressure, P_e the exit pressure, K_1 is an experimentally determined constant for the inlet restriction, Q_1 is the mass velocity into the surge tank and Q_2 is the mass velocity out of the surge tank (compressible volume). The first equation represents the pressure-drop across the inlet restriction and is a statement of the momentum equation across the restriction. Note that the surge tank is downstream of the restriction. The second equation is the pressure-drop between the surge tank and the system exit

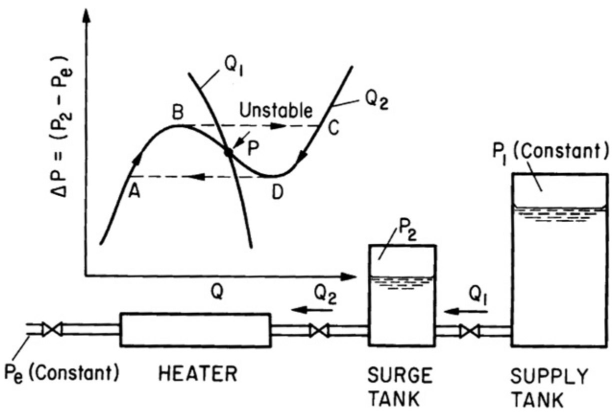

Fig. 3. Schematic flow diagram and limit cycle of pressure-drop type oscillations.

section, and it is the system curve shown in Fig. 3. While operating on the negative slope portion, a slight increase in the surge tank pressure would cause more fluid to enter it than leaves it. The surge tank pressure now increases due to accumulation of fluid. The operating point moves up until it reaches the peak (point B). Any higher pressure can be sustained only by a higher mass flow rate as given by the system curve. This point is found to be in the single-phase liquid region (point C). At C, the amount of fluid leaving the surge tank is more than the amount entering it. Therefore, the surge tank pressure decreases till the operating point reaches the curve minima at D. Any lower pressure can now be obtained only if the mass flow rate reduces to the value at point A. Hence, an excursion to A is observed. Now the mass leaving the surge tank is less than that enters it. Hence pressure goes up pushing up the operating point till A is reached, where once again a flow excursion is observed. Thus perturbation at any point on the negative slope region results in a flow oscillation tracing the limit cycle ABCDA. This is essentially the mechanism of pressure-drop oscillations. Note that they can occur only if the system possesses a compressible volume, either external or internal. In actual two-phase heat exchange equipment there is a presence at certain times of some internal compressible volume which can trigger such oscillations. Typical recordings of the pressure-drop oscillations observed are given in Fig. 8. Internal characteristics with negative slopes, and external characteristics steeper than the internal characteristics as well as the existence of a compressible volume (e.g. surge tank) in the flow circuit are the conditions necessary for the occurrence of the pressure-drop type oscillations.

3. Approaches in two-phase flow stability analysis

Two-phase flow stability analysis, as the stability analysis in general, is a field of importance in science and technology. The stability analyses of any physical system has always been facilitated by achievements in mathematics. In this section, we take a look at the major approaches to investigating the stabilities or instabilities encountered in boiling two-phase flows [12].

Direct numerical analysis. Direct numerical analysis is often called time-domain analysis, or nonlinear time domain finite-difference method, in which the conservation equations for two-phase flow are solved directly by numerical methods. In direct numerical analysis, if the solutions converge to the original equilibrium, the system is said to be stable; otherwise, if the solutions diverge or oscillate, the system is said to be unstable.

Frequency-domain analysis. Frequency-domain analysis was first developed and applied in control theory. The method uses Laplace transformations to determine the location of the roots of the characteristic equation in the complex plane. If any of the roots have a positive real part, we conclude that the state is linearly unstable. If a lumped-parameter description of the system under investigation is chosen, generally a system of ordinary non-linear differential equations, which are first-order in time, will be obtained. To determine for what parameter values the system is stable one needs to find the root with the largest real part for each value of the flow parameters. Root finding can be done with a numerical method. The analysis can also be done in the frequency-domain by using the Nyquist criterion. Whichever technique is chosen, the procedure must be carried out for a sufficient large number of parameter values to obtain the stability threshold in parameter space. If a distributed description of the system is chosen, a system of partial differential equations will be obtained. Since the Laplace transformation of partial differential equations does not result in algebraic equations, some account must be taken of the spatial variation. This can be done by discretization in space, numerical integration on the spatial part of the differential equations, or direct solution of the partial differential equations by integrating along the characteristics.

3.1. Dynamical system analysis

The study of dynamics has been one of the most fascinating fields in physical science for thousands of years. A well known branch in this field is celestial mechanics, especially the study of motions of the bodies in the solar system. Newton's attempts to understand and model the observed motions of these bodies incorporated Kepler's laws and led to his development of calculus. With this the study of models of dynamical problems as differential equations began. In spite of the great elegance and simplicity of such equations, the solution of specific problems proved remarkably difficult and engaged the minds of many of the greatest mechanicians and mathematicians of the eighteenth and nineteenth centuries.

While a relatively complete theory was developed for linear ordinary differential equations, non-linear systems remained largely inaccessible, apart from successful applications of perturbation methods to weakly non-linear problems. Poincare [13,14] was the first one to view an ordinary differential equation as defining a family of orbits, thereby establishing the geometric or qualitative theory of dynamical systems. The aim of the dynamical system theory is to understand the time-asymptotic behavior of the solutions of a given system in terms of the geometric and topological structures in the phase-space. Thus the theory relies on visualization of 'solutions flows' in the phase-space. The long-term behavior of the solution-flow is classified in terms of attractors, which are phase-space structures. The three classic attractors are equilibrium, periodic motion or limit-cycle, and quasi-periodic motion. These are called attractors because if some form of damping is present, which is generally true of all real-life systems, the transients decay, and the system is attracted to one of these structures.

Besides Poincare [13,14], Liapunov [15], Birkhoff [16], Andronov and co-workers [17–19] and Arnold [20–23] have made outstanding contributions to this field. Though the theoretical development in the field of non-linear dynamic systems has been very rapid, until the mid-1970s the new tools were largely in the hands of pure mathematicians.

Following the pioneering work by Lorentz [6], there has been growing interest in the applied science community. Then, there has been an explosion of interest in the study of non-linear dynamic systems. Scientists in all disciplines have come to realize the power and the beauty of the geometric and qualitative techniques. The field of non-linear dynamic systems has been hailed as the third scientific revolution of the twentieth century, after Relativity and Quantum Mechanics [24].

Some pioneering work has been carried out in two-phase flow dynamics using the tools in non-linear dynamical systems. Archard et al. [25] performed Hopf bifurcation analysis on non-linear density-wave oscillations in boiling channels. Rizwan-uddin and Doming [26,27] conducted non-linear numerical simulations of density-wave oscillations. Ozawa et al. [28] have used the Liapunov method to determine the stability of their parallel-channel system against pressure-drop type oscillations. Zhang et al. [29] developed an explicit criterion for density-wave instability by using the Liapunov method. Padki et al. [30] analyzed pressure-drop type and Ledinegg instabilities by the Hopf bifurcation method.

4. Researches in two-phase flow instabilities

There have been several extensive reviews of instabilities in two-phase flow systems, notably by Boure et al. [2], Bergles [4,5], Yadigaroglu [31], Ishii [32], Lahey [3] and Kakac and Liu [33]. There are many types of two-phase flow instabilities as shown in Table 1, and the research activities have been so extensive that a complete account of them becomes infeasible. Akagawa [34] presented research work on two-phase flow in universities and colleges in Japan. He tabulated Japanese research work on flow instabilities has also been presented by Nakanishi [35]. We now proceed to dis-

Table 1
Classification of boiling two-phase flow instabilities

Class	Type	Mechanism	Characteristics
Static instabilities	Ledinegg instability	Internal characteristics steeper than external characteristics in a negative slope region	Flow undergoes sudden, large amplitude excursion to a new stable operating condition
	Boiling crisis	Ineffective removal of heat from heated surface	Wall temperature excursion and flow oscillation
	Flow pattern transition instability	Bubbly flow has less voidbut high pressure-drop than annular flow	Cyclic flow pattern transitionand flow rate variations
	Bumping Geysering Chugging	Periodic adjustment of metastable condition,usually due to lack of nucleation sites	Periodic process of super-heatand violent evaporation with possible expulsion and refilling
Dynamic instabilities	Acoustic oscillations	Resonance of pressure waves	High frequencies (10–100 Hz) related to time required for pressure wave propagation in System
	Density-wave oscillations	Delay and feedback effects in relationship between flow rate,density, and pressure-drop	Frequencies related to transit time as a continuity wave
	Pressure-drop oscillations	Dynamic interaction between channel and compressible volume	Very low frequency periodic Process
	Thermal oscillations	Interaction of variable heat transfer coefficient with flow dynamics	High magnitude temperature oscillations in the solid due to transitions between different boiling regimes
	Boiling water reactor instability	Interaction of void reactivity coupling withflow dynamics and heattransfer	Strong only for small fuel timeconstant and under low pressure
	Parallel channel instability	Interaction among parallel channels	Various modes of flow redistribution or V-tube manometer oscillations

cuss the main types of instability encountered in boiling two-phase flow systems which are Ledinegg instability, density-wave, pressure-drop, and thermal oscillations.

4.1. Ledinegg instability

Maulbetsch and Griffith [36] studied Ledinegg instability in a subcooled boiling system with a large bypass channel. With constant pressure-drop boundary condition, excursions leading to critical heat flux (CHF) were always observed near the minima on the internal characteristic curve. Ledinegg and Wiesenberger [37] offered further discussions of the instability encountered in parallel channel systems. Akagawa et al. [38] experimentally studied Ledinegg instabilities. The experiment used a three parallel channel evaporator tube system charged with R-113. The data had a high ratio of length-to-diameter for the test section. The experimental results were presented in graphical forms. Margetts [39] reported excursive instability observations in full-scale feed water economizers. Boiler feed water was preheated in a large number of parallel channels by recovering heat from ammonia plant primary flow gases. Calculations suggested that the severely over-heated tube had the highest head load and was operating at the minima in the pressure-drop versus flow rate curve, which was essentially flat and set by the remaining parallel tubes in the bank. The problem was solved by eliminating one-third of the unheated tubes to increase the driving pressure-drop of the heated tubes. Lorezini [40] presented a simplified method for practical determination of the Ledinegg instability. The instability phenomenon was outlined and two methods were provided for the determination of the existence of the Ledinegg instability, and if it exists, how to eliminate it.

4.2. Density-wave oscillations

Experimental researches on density-wave oscillations. Density-wave oscillation is by far the most studied type of oscillation in two-phase flow instability problems, and the amount of published experimental work in this field is overwhelming. In this subsection, we provide a survey of some major experimental work in this field. For convenience, a chronological order will be followed without any implied indications. Semenovkel [41] reported the results of experimental studies on two-phase flow instabilities in the Soviet Union during the 1950s. Oscillations were investigated using a multi-channel circulation boiler. Unstable conditions were set by either decreasing the flow rate or by increasing the heat input. Oscillations were observed in all parallel channels with some phase-shift among the channels. Introduction of additional pressure-drop at the inlet (smaller diameter orifices were employed for this purpose) stabilized the system. Special experiments were conducted to find the effect of multi-valued characteristics of the system pressure-drop curve, and it was concluded that multi-valuedness had no effect on the stability of the system. Davidov [42] investigated the flow oscillations using electrically heated tubes. Oscillations with periods roughly equal to the transit time of a fluid particle in the heater were observed. He found out that interconnecting the channels and increasing the relative size of the subcooled length had a stabilizing effect, while exit throttling had a destabilizing effect. Wallis and Heasley [43], Anderson et al. [44] and Jain [45] studied the effect of inlet and exit restrictions. They found that an inlet restriction, which increased single-phase flow friction, had a stabilizing effect, while an exit restriction, which increased the two-phase friction, reduced the flow instability. Stenning [8], Stenning and Veziroglu [3] identified three distinct two-phase flow instability modes, and they first used the term "density-wave oscillations." The term has been widely adopted later. They provided a simple explanation of the origin and mechanism of density-wave oscillations. Crowley et al. [46] studied the effect of channel length with a constant power density in a Freon loop. By cutting out a section of heated length at the inlet and restoring the original flow rate, they found that the reduction of the heated length increases the flow stability in forced circulation. Mathisen [47] found a similar effect in a natural circulation loop. Friedly et al. [48] studied the stability in a once-through liquid-nitrogen evaporator for a fixed inlet subcooling, and under the conditions that the system pressure-drops were dominated by inlet and exit valves. Three definite regions were observed on the plot of the inlet to exit pressure-drop ratio versus the exit quality. System stability decreased with the decrease in the pressure-drop ratio. On the other hand, the amplitude of the oscillations consistently increased with increasing pressure-drop ratios. Veziroglu and Lee [49–51] investigated density-wave oscillations in two parallel channels with and without cross-connections, using R-11 as the test fluid. For the case without cross-connections, they observed that the density-wave oscillations occurred simultaneously in both of the parallel channels and were out of phase by 180 degrees. In contrast, for the cases with cross-connections, the density-wave oscillations were in phase. It was possible to stabilize the cross-connected system by introducing a pressure-drop at the inlet, but this was not possible for the system without the cross-connections. The system with cross-connections was found to be more stable than both the system without cross-connections and the single channel system. Collins and Gacesa [52] performed experiments to study the effect of the by-pass ratio on density-wave instability in parallel channel flow. They tested the power at the threshold of flow oscillation in a 19 rod bundle by changing the by-pass ratio from 2 to 18. They found that a high by-pass ratio destabilized the flow in parallel channels. Collins et al. [53] described unstable behavior in a full-scale simulated nuclear reactor. As bundle power was increased to the point where low quality steam was generated, a variety of oscillations in flow rate were observed. While it appears that both density-wave and pressure-drop oscillations were present, the major fluctuations were definitely of the density-wave variety. Detailed measurements of the threshold of periodic dryout were recorded as a function of system geometry and flow conditions. Yadigaroglu and Bergles [54] also observed higher order oscillations in their experimental studies. They found that the frequencies of higher order oscillations were the integral multipliers of the frequency of lower order oscillations. They observed that a high-frequency oscillation occurred prior to a low-frequency oscillation. The presence of the higher modes was explained in terms of the dynamic behavior of the boiling boundary. Kakac et al. [55,56] studied both sustained and transient boiling flow instabilities in a cross-connected four parallel channel up-flow system. Both R-11 and R-113 were used as test fluids. They reported that density-wave oscillations occurred simultaneously in all of the four channels and they are in phase. Saha et al. [57] carried out an experimental study on the onset of density-wave oscillations using a uniformly heated boiling channel. They studied the effects of inlet subcooling, system pressure, inlet and exit restrictions and the inlet velocity. Veziroglu et al. [58] investigated the stability boundaries of sustained density-wave oscillations in an electrically heated single channel, up-flow system with R-11 as the test fluid. In the range of their experiments, the density-wave oscillation boundaries showed an almost linear relationship between the degree of subcooling and the mass flow rate. There was departure from the linear relationship at higher inlet subcoolings. Aritomi et al. [59] presented the results of experimental studies of density-wave type oscillations in a parallel-channel system using water as test fluid. They studied the nature of oscillations and stability boundaries under various conditions of inlet velocity, heat flux, liquid temperature, cross-section of the channel and entrance throttling. Ozawa et al. [60,61] studied density-wave oscillations in both single and parallel multi-channel systems. They compared the results for single and multi-channel systems to find out the similarities and differences. Density-wave oscillations were observed at the positive slope region of the pressure-drop versus flow rate curve, and flow instability maps were generated for both single and multi-channel systems. France et al. [62] performed density-wave stability experiments using high pressure boiling water and liquid heating. Stability thresholds were determined and a correlation was proposed. Xu and Chen [63] conducted an experimental investigation of density-wave type instability in a vertical up-flow system. The fluid used was water and the pressure was at medium–high and the heat input was uniform. The closed cycle of total system pressure-drop versus mass flow rate and oscillation curve were obtained. The effects of various parameters on stability boundaries and oscillation periods were studied in a wide range of parameters. Wang et al. [64,65] investigated density-wave type oscillations in high pressure water loops. The effects of system parameters on density-wave oscillations were studied in a wide range. They also correlated empirically the experimental results for the threshold of density-wave oscillations. Gao et al. [66] studied the stability in a low pressure, low quality two-phase flow with natural circulation. They indicated that because of the coupling between the thermal-hydraulic and neutronic parameters, the density-wave oscillations in the two-phase natural circulation may intensify.

4.2.1. Theoretical researches on density-wave oscillations

In one of the earliest models of density-wave oscillations, Wallis and Heasley [43] integrated the energy equation along the heated channel, adopting the Lagrangian approach. They assumed that the pressure-drops were concentrated at the inlet and exit, and integrated the momentum equation to obtain the characteristic equation of the channel. The equation was then analyzed using the Nyquist locus plot. Quandt [67] conducted both experimental and analytical stability studies on parallel channel systems. He distinguished two types of parallel channel instabilities, namely flow excursions and flow oscillations. Assumptions were made regarding the spatial form of the flow and enthalpy perturbations such that the perturbation equations could be integrated in the flow direction and then Laplace transformation could be performed. The criteria for flow instabilities could be determined from the transfer function in terms of the initial fluid conditions, channel geometry, and heat flux distribution. Meyer and Rose [68] used the Eulerian approach to formulate the conservation equations, and solved these equations in time domain using finite differences. The system under consideration was a boiling channel with constant pressure-drop across it. This was one of the earliest computer codes which solved two-phase flow equations directly. The computer code, STABLE, was developed by Jones and Dight [69–72] for the prediction of the onset of density-wave oscillations in a boiling channel. A constant pressure-drop boundary condition was assumed to be imposed across the channel as the feedback mechanism of the density-wave oscillations. The equations of conservation of energy, mass, and momentum were applied in the subcooled and bulk boiling regions of the channel. Boundary conditions were specified at the interface between the two regions and at the channel entrance and exit. A set of linearized and Laplace-transformed differential equations was obtained and integrated along the channel. The results represented the transfer function for each spatial node. System stability could then be evaluated by techniques of control theory, specifically the Nyquist method. Jones and Yarbrough [73,74] incorporated the nuclear feedback into the STABLE code and developed the computer code FABLE for the nuclear-coupled density-wave oscillations. Davies and Potter [75] analyzed the unstable flow in parallel channels using one-dimensional homogeneous flow model. Linearization and Laplace transformation were used to develop a set of transfer functions so that the flow instability problem could be represented as a feedback system shown in a typical feedback loop block diagram. A computer code, LOOP, was developed to evaluate the transfer functions and to determine the responses of the feedback system. Linear feedback theory, e.g. Nyquist criterion, pole position, was used to assess stability. Efferding [76] developed the code DYNAM as a design tool for the study of flow oscillations in once-through steam generators. Jone's model in the STABLE code was selected as the representation of subcooled and bulk boiling regimes. The complete code represents four heat transfer regimes present in a once-through steam generator, i.e. subcooled convection, subcooled nucleate boiling, bulk boiling, and superheated. Zuber [77–79] carried out theoretical studies to determine the fundamental nature of oscillations and instabilities in the flow of cryogenics with heat addition. Zuber was the first to formulate the two-phase flow instability problem in terms of the center of mass and to take into consideration the effect of relative velocity between the two phases by the drift-flux flow model. The formulation consisted of the continuity, energy and momentum equations for the mixture, the continuity equation for the vapor phase, and seven constitutive equations. By using the small perturbation technique, the set of equations for a system in thermal equilibrium with a constant heat flux was integrated analytically, resulting in a characteristic equation for a distributed-parameter system. Ishii and Zuber [80] generalized Zuber's formulation [77–79] and obtained an important similarity group, i.e. subcooling number, phase change number, drift number, etc. The system stability boundary was obtained in the plane of subcooling number versus phase change number. A simple stability criterion for high sub cooling number was also obtained. Later, Saha et al. [57] included the effect of thermal non-equilibrium between the phases in the stability analysis of a heated boiling channel by introducing a constitutive equation for the mass rate of vapor from a steady-state energy consideration. Crowley and Bergles [81] performed the fluid-to-fluid modeling of density-wave oscillations. Application of scaling parameters obtained from the analytical formulation led to the conclusion that good similarity could be achieved only by using a full scale model. However, the power and the pressure level requirements were much lower than those of the prototype. So the modeling is appropriate only if the test section pressure-drop is small compared with the pressure level of the system. So they suggest an alternate modeling procedure based on similarity of the ratio of pressure-drop to system pressure. Takahashi and Shindo [82,83] tried to explain the mechanism of density-wave oscillations theoretically, and to determine the effect of different parameters. They used slip-flow model for their analysis and employed empirical correlations for the heat transfer and the pressure-drop calculations. Their first results indicated that the heat exchange between the fluid and the channel wall, which alternately stores and releases heat, was the main factor for the oscillations. However, their theoretical work using conventional linear control theory which simplified the heat transfer model, indicated that the effect of the fluid column height was greater than that of heat transfer characteristics. Yadigaroglu and Bergles [54] investigated the stability of a boiling channel with regard to density-wave oscillations by oscillating the inlet flow. They observed that the oscillation characteristics were uniquely determined by the enthalpy perturbations at the steady-state location of the boundary. Because of this, the delay of the enthalpy perturbation at the average position of the boundary was of primary importance for the channel stability. At high subcooling and low heat input experiments with R-l13, they observed higher modes of oscillations with frequencies multiple of fundamental frequency. Lahey and Yadigaroglu [84] developed a computer program, NUFREQ, to predict the onset of density-wave oscillations in adiabatic two-phase systems, such as BWR. The method of characteristics was employed to solve the conservation equations in a Lagrangian frame of reference. An analytical solution of the non-linear system could then be obtained. Heater wall dynamics, boiling boundary dynamics and nuclear reactor kinetics were included in the model. Moreover, the simplicity of this model provided valuable insight into the fundamental mechanisms of density-wave oscillations in two-phase flow systems. Nakanishi et al. [85,86] studied density-wave oscillations in a single channel boiling system analytically using the small perturbation method. The heat capacity of the wall in the subcooled region and the effect of relative velocity between the phases in the two-phase region were taken into account. Takitani [87] developed a linear lumped-parameter transfer function model to analyze density-wave oscillations. In the analysis, the heater was treated in three distinct regions: subcooled, saturated and superheated regions. Heat flux was assumed to be steady, except in the subcooled region, where the dependency of the heat flux on flow rate was taken into account. The pressure losses in the saturated and superheated boiling regions were neglected. With these assumptions, characteristic equations with three time constants were obtained.

Akyuzlu et al. [88] investigated pressure-drop type and density-wave type oscillations theoretically and experimentally, and the theoretical results were verified by the experimental findings. Fukuda and Hasegawa [89] and Fukuda et al. [90] studied analytically the modes of oscillations using the characteristic equation for a parallel channel system. Each channel was described in terms of a transfer function, and the characteristic equation was formed by combining all the channels. They found that the real part of the root of the characteristic equation which corresponds to the out-of-phase mode was always larger than that for the in-phase mode. This result agrees with the experimental finding that the out-of-phase oscillation mode is easier to realize than the in-phase one. Achard et al. [25,91] presented the results of linear and non-linear analytical investigation into thermal-hydraulic instability modes in boiling channels. They showed that when frictional and gravitational effects were properly considered, "islands of instability" may occur in linearly stable regions. They further used the Hopf bifurcation theorem to prove the existence and uniqueness of density-wave oscillations. The non-linear analysis indicated that disturbances of finite amplitude in the flow could lead to precipitous instabilities for conditions that were linearly stable (subcritical bifurcations), while limit-cycle oscillations of small amplitude were possible for conditions that were linearly unstable (supercritical bifurcations). Yokomizo [92] developed an equivalent linearization technique to analyze two-phase flow oscillations with finite amplitude. A one-dimensional slip-flow model was used to formulate the linearized equations in the frequency-domain, and the equations were numerically solved. Good agreement with a detailed linear frequency-domain program was reported for very small amplitude oscillations. Limit-cycle amplitudes were also calculated from the knowledge of the amplitudes at the stability boundary. Rizwan-uddin and Doming [26] numerically simulated the non-linear dynamics of two-phase flow in heated channels. The parallel-channel density-wave analysis was extended to the case of a simplified loop which includes the heated channel and the pump characteristics. Existence of stable limit cycles was found in the linearly unstable region close to the marginal stability boundary. Sensitive dependence upon initial histories and initial conditions was found for parameter values for which there existed more than one attracting set: stable fixed point and stable limit cycle. Rizwan-uddin and Doming [27] investigated chaotic attract in a periodically forced two-phase flow system. Motivated by the enhancement of heat transfer under oscillation flow conditions in single-phase flows, they analyzed density-wave oscillations by solving the non-linear, variable-delay, functional, ordinary integro-differential equations which resulted from integrations of the non-linear partial differential equations. They studied three cases of forced flow: constant pressure-drop across the channel, exponentially decaying pressure-drop and periodic pressure-drop. It was found that for a periodic pressure-drop condition, a strange attractor existed. Khabenskii and Kvetnyl [93] used the results of numerous experimental and theoretical researches that had been carried out by both the former Soviet and Western scientists to reveal principles by which operational and design parameters affect the boundary of oscillatory instability of two-phase flow. They presented simplified methods for making a preliminary evaluation of the boundary of thermohydraulic instability in a channel with constant pressure-drop across it. Sami and Kraitem [94] developed a computer program DYNAM/US for predicting thermally induced flow oscillations in a once-through boiling flow with superheat. The model included newly developed dynamic schemes for calculating subcooled, transition and bulk boiling, as well as superheat. Numerical results compared well with experimental data, and the model was able to predict the lower natural frequencies, as well as the system frequency response at higher frequencies. Yang et al. [95] investigated two-phase flow instabilities in a vertical U-tube evaporator. A mathematical model describing the flow oscillations was developed and numerically integrated. The calculated results were found to be in good agreement with the experimental data. Clausse et al. [96] presented an analysis of stability and oscillation modes in a boiling multi-channel loop using the theory of perturbation of parameter dependent linear operators. This theory permits the analysis of asymmetric systems. Simple analytical criteria were found which greatly reduce the complexity of the analysis required for coupled parallel channels. Zhang et al. [2] analyzed density-wave oscillations using an energy principle based on Lyapunov's second method. The general excess entropy production criterion was introduced as a general energy principle in the field of two-phase flow instability analysis. They discussed the sufficient and necessary conditions of the energy principle and a comparison of stability boundaries estimated by the energy principle and linear frequency-domain method was conducted. Xiao et al. [97] presented experimental results and analytical modeling of density-wave type oscillations in parallel boiling channels under high pressure. They observed different types of two-phase flow instabilities, including density-wave type, pressure-drop type, thermal and second density-wave oscillations. A multi-variable linear model was developed to analyze the system stability in the frequency-domain by means of multi-variable control system theory. The multi-channel boiling system was expressed by transfer matrix models which included feedback on the two-phase pressure-drop and external loop pressure-drop. Delmastro et al. [98] studied density-wave instabilities in boiling channels based on delay equations derived from the homogenous two-phase flow conservation equations. The character of the oscillatory instabilities was studied using Hopf perturbation methods.

4.2.2. Pressure-drop type oscillations

The generally accepted term, "pressure-drop type oscillations," was proposed by Stenning [8] and Stenning and Veziroglu [9]. In these works, they observed three distinct modes of flow oscillations, having termed these types as "density-wave," "pressure-drop," and "thermal" oscillations. It was pointed out that the pressure-drop oscillations occurred only when the pressure-drop across the test section decreased with increasing flow rate, and the oscillation period was governed by the volume and compressibility of the vapor in the system. Stenning et al. [10] carried out both experimental and analytical studies of pressure-drop type oscillations in forced convective boiling systems with R-11 as the test fluid. They developed a linearized model to predict the threshold of oscillations. Maulbetsch and Griffith [36] reported both experimental and analytical studies of pressure-drop type oscillations in forced convective flows with subcooled boiling. They described pressure-drop type oscillations resulted from the interaction between the heated section and another energy storage device in the loop, usually a compressible volume. The steady-state pressure-drop versus flow rate characteristic curve was used for the heated section, and a lumped model was used for the compressible volume. Veziroglu and Lee [99] investigated pressure-drop type and density-wave type oscillations in a single vertical boiling channel using R-11 as the test fluid.

They compared the results with those of a horizontal channel system, and found that the vertical upward flows were more stable than the horizontal flows. Veziroglu and Lee [49–51] also studied the instabilities in two parallel channels with and without cross-connections. In the case without cross-connections, they observed that the pressure-drop oscillations could occur in either one or both of the channels, and they were always in phase when they occurred in both channels simultaneously. In contrast, with the cross-connections, pressure-drop type oscillations always occurred simultaneously in both of the channels. The system with cross-connections was found to be more stable than both the system without cross-connections and the single channel system. Kakac et al. [55,56,100] investigated sustained and transient boiling flow instabilities in a cross-connected four parallel channel up-flow system using R-11 and R-113 as test fluids. They reported that pressure-drop type and density-wave oscillations occurred simultaneously in all the four channels and they were in phase. Ozawa et al. [60] theoretically studied pressure-drop type oscillations in boiling channels by suing a lumped parameter model. In this study, the pressure-drop characteristics were approximated by a third-order equation in the flow rate. With an additional implied assumption, the momentum equation in the boiling channel was reduced to a Van der Pol equation. Though the validity of the implied assumption is very questionable, this work pioneered the applications of non-linear ordinary differential equation theories to the analysis of pressure-drop type oscillations. Ozawa et al. [61] also studied pressure-drop type oscillations in a gas–liquid two-phase flow system. They conducted an experimental investigation of pressure-drop type oscillations in a system with a negative slope region in the pressure-drop characteristics and compressible volumes in both the gas and liquid supply lines. In the modeling approach, a similar method used by Ozawa et al. [60] was employed, and a Van der Pol equation was also obtained. Mentes et al. [101] studied the effect of different heater surface configurations on two-phase flow instabilities in a single channel up-flow system. R-11 was used as the test fluid, and six different heater tubes with various inside surface configurations were tested. Experimental results were presented on system pressure-drop versus mass flow rate curves, on which stability boundaries were indicated. Tubes with internal springs were found to be more stable than the other tubes. Gurgenci et al. [102] developed a constant-property homogeneous-flow model to generate the limit cycles of pressure-drop and density-wave oscillations in a single-channel upflow boiling system operating between constant pressures. The upstream compressible volume was introduced through a surge tank. In the model, thermodynamic equilibrium conditions were assumed and the effects of the wall heat storage and the variation of the fluid properties were neglected. For pressure-drop type oscillations, the simple model produced fairly good results compared with their experimental findings. For the density-wave oscillations, the theoretical frequencies were in good agreement with the experimental frequencies, though the amplitudes were not. Gurgenci et al. [103] also developed a criterion for linearized stability for both the pressure-drop and density-wave oscillations for the same single-channel upflow system. Two different two-phase flow models, the constant-property homogeneous flow model and the variable-property drift-flux flow model, were employed. The conservation equations for both models and the equation of surge tank dynamics were first linearized for small perturbations and the stability of the resulting set of equations for each model was examined using Nyquist plots. As a measure of the relative instability of the system, the level of inlet throttling necessary to stabilize the system at particular operating points were also calculated. The results were compared with experimental findings. Comparisons showed that the drift-flux formulation offered a reliable way of determining the instability threshold. Lin et al. [104,105] reported results of a theoretical study on the modeling of pressure-drop oscillations in a horizontal hair-pin tube. The flow was assumed to be homogeneous, with thermodynamic equilibrium between the phases. The conservation equations for mass, momentum, and energy were applied to one-dimensional viscous, compressible fluid flow. Numerical solutions of these equations were obtained and satisfactory predictions were reported. Ozawa et al. [28] studied pressure-drop oscillations in an air–water flow parallel-channel system with compressible volumes. They reported two kinds of pressure-drop oscillations: relaxation oscillations and quasi-static oscillations. The modes of oscillations observed in the parallel-channel system were a single-channel mode, a U-tube mode and a multi-channel mode, and the modes were closely tied to the type of oscillations. The non-uniformity of flow distribution was analyzed. A stability analysis was also carried out using the Lyapunov function approach. Wang and Chen [64] presented experimental results of pressure-drop oscillations in a medium–high pressure system. They studied the effect of different ratios of channel-to-exit restriction diameters, and found that the oscillations occurred only when the diameter-ratio was sufficiently high. As this ratio increased, the magnitude of the negative-slope also increased, making the system more unstable. Padki et al. [106] reported results of both theoretical and experimental investigations of pressure-drop type and thermal instabilities in forced convection boiling system. Under the experimental conditions of the study, pressure-drop type and thermal oscillations, in addition to pressure-drop type with superimposed density-wave oscillations, were observed. A numerical model, based on the drift-flux model, was developed to predict the steady-state characteristics of the forced convective two-phase flow and the pressure-drop type and thermal oscillations. Good agreement between the theory and experiments was reported. Padki et al. [30] performed the first bifurcation analysis on the pressure-drop oscillations and Ledinegg instability from the perspective of dynamical system theory. An integral formulation was developed to model the two-phase flow system. Instability criteria independent of the actual two-phase flow model were derived for both the pressure-drop type oscillations and Ledinegg instability. It was shown that the pressure-drop oscillation limit-cycles occurred after a super-critical Hopf bifurcation. In an extension of the analysis, an effort was made to clarify the mechanisms of the pressure-drop type oscillations and Ledinegg instability. Liu et al. [107] made a dynamical analysis of pressure-drop type oscillations with a planar model. In this study, it has been proven theoretically the existence, the uniqueness and the stability of the limit-cycle of pressure-drop type oscillations and provided the whole bifurcation diagram of the dynamic system [108]. Cao et al. [109] presented a theoretical model for the two-phase flow pressure-drop type instabilities in an upflow boiling system including the thermal non-equilibrium effect between the two phases. The modeling results were verified by experimental findings. Widman et al. [110] and Karsli et al. [111] investigated experimentally the effect of augmented surfaces on two-phase flow instabilities in a horizontal in-tube boiling system with three tubes made of different surfaces at five different inlet temperatures. Comakli et al. [112] made an experimental study to investigate two-phase flow dynamic instabilities in a horizontal in-tube boiling system. Mawasha and Gross [113] investigated a horizontal boiling channel with a surge tank through non-linear analysis which also included the effect of the wall thermal capacity. Guo et al. [114] conducted experiments in a high pressure steam-water two-phase flow in a helical tube to study pressure-drop type oscillations. A series of correlations are proposed for the average and local heat transfer coefficients under oscillatory conditions.

4.2.3. Thermal oscillations

The term "thermal oscillations" has been used in several different situations where temperature fluctuations occur in a solid interacting with a fluid. The interpretation of the thermal oscillations used here is related to the instability of boiling heat transfer, and was first identified by Stenning and Veziroglu [9]. The thermal oscillations are characterized by large amplitude fluctuations in the heater wall temperature. The flow oscillates between annular flow, transition boiling and droplet flow at a given point and thus produces large amplitude temperature oscillations. Density-wave oscillations are required to trigger the thermal oscillations. They observed thermal oscillations in a single channel convective boiling system. In this paper, they pointed out that density-wave oscillations were necessary to trigger the thermal oscillations. It was also stated that thermal oscillations were associated with film boiling conditions and produced large changes in heater surface temperature. Pakopoulos et al. [115] carried out a study on the dynamics of two-phase flow in vapor generators, focusing on the effect on density-wave instability of the coupling between heat flux and flow. A distributed-parameter, transient theoretical analysis of the conservation equations of two-phase flows was presented. The dynamics of the heater wall was taken into account and the prominent role of the heat flux and flow rate coupling was highlighted. Kakac et al. [116,117] reported both experimental and theoretical investigations on thermal oscillations in a forced convection upflow boiling system. The thermal oscillations studied were induced by pressure-drop type oscillations. Experiments with various heat inputs and inlet fluid temperatures, as well as for different heater tube sizes, were conducted to investigate the effects of these parameters on the thermal oscillations. In the theoretical study, one-dimensional governing equations have been solved numerically to determine the wall conditions, fluid properties, and flow conditions at any point along the test section. The model was based on the assumption of homogeneous two-phase flow, with thermodynamic equilibrium between the phases. The conditions under which thermal oscillations occurred were predicted. Liu [12] and Liu and Kakac [33] presented an experimental study of the thermal oscillations in a convective boiling upflow system. The thermal oscillations studied were accompanied by pressure-drop type oscillations, and the pressure-drop type oscillations were responsible for inducing the thermal oscillations. The effects of mass flow rate, heat input, inlet liquid temperature and upstream compressible volume on the thermal oscillations were studied. Results were presented in tabular and graphical forms. Liu et al. [118] studied the characteristics of transition boiling and thermal oscillations in a single-channel forced convection upflow system experimentally. It was found that in the convective transient boiling region, the wall temperature fluctuations have two distinct modes, one with high frequency and small amplitudes and the other with low frequency and large amplitudes.

5. Experimental systems

This section introduces the basics of the experimental set-ups by several researchers used in the study of two-phase flow dynamic instabilities. In the experimental study, three different experimental set-ups were used namely, vertical and horizontal two-phase flow loops (Figs. 4 and 5). They were designed and built to generate the pressure-drop type, density-wave type and thermal oscillations, and to investigate the effects of inlet sub cooling, heat flux, flow rate and upstream compressible volume on the system stability [12,120].

Different fluids were used as the test fluid due to its relatively low boiling point and latent heat of vaporization as well as its excellent scaling similarity with water. The test fluid, R-11, was supplied from the main tank pressurized by nitrogen gas. A cooling unit before the test section provided an inlet temperature range of −20 °C to 25 °C, with a control accuracy of ±1.0 °C. Following the electrically heated test section was a recovery section consisting of a condenser and a collector tank. The mixture of saturated liquid and vapor was directed through the condenser coil. The condensed liquid was then stored in a recovery tank which was maintained at a constant pressure. This arrangement ensured a constant pressure level in the container and

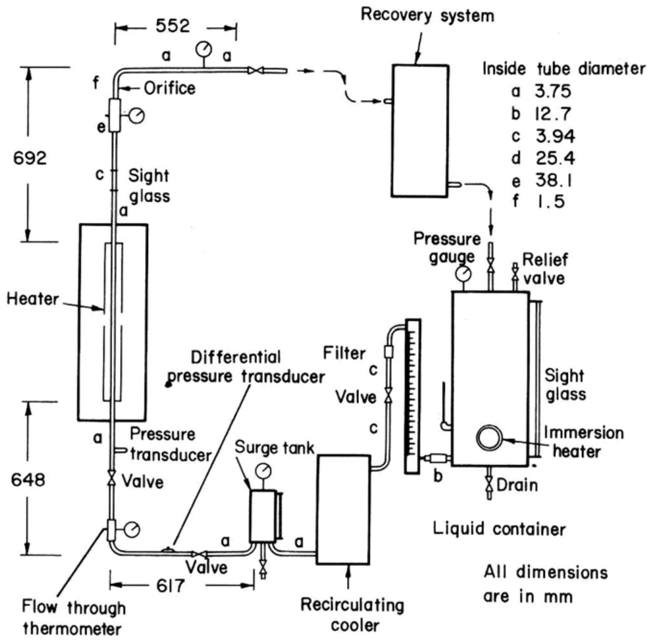

Fig. 4. Schematic diagram of vertical boiling flow system.

at the exit. A full size high pressure water-vapor boiling system as shown in Fig. 6 was also built and used to investigate the basic mechanisms of different types of oscillations and the effects of various parameters on these oscillations using degassed water.

Appropriate instrumentation was installed to control and measure the test parameters, namely flow rate, temperature and pressure at various locations, and the electrical heat input. The experimental set-up can be divided into three main segments: the fluid supply section, the test section, and the recovery section. *The fluid supply* section was comprised of the main tank, a filter, a rotameter, and a subcooling controller. A high pressure nitrogen supply tank connected to the main tank provided the necessary driving force to pump the liquid through the system. *The test section* was the part of the loop where the boiling and related two-phase flow oscillations occurred under controlled test conditions. It included a surge tank, a heater, two plenums and a riser, followed by an exit restriction. The main body of instrumentation was also clustered in this section. Following the test section was the *recovery section* consisting of a condenser and a recovery tank. The mixture of liquid and vapor discharged from the exit restriction was directed through the condenser coil, which was cooled by refrigerated brine. The condensed liquid was then collected in the recovery tank.

5.1. Experimental procedures

For a given heater tube, different sets of experiments corresponding to various heat inputs and different inlet liquid temperature were conducted. Each set was composed of a sufficient number of tests to cover the available flow range. Stability boundaries were determined in each case. Oscillations were identified by the cyclic variations in pressures, wall temperature and flow rates, and by observing the pressure-gauge pointers, and recordings. In defining the stability boundary, short-life transients were disregarded and only the sustained oscillations were considered. Experiments with different upstream compressible volume were also conducted.

The test procedure is best summarized in the following steps [65,119,121]: (1) With enough liquid in the main tank and inlet temperature set, the tank is pressurized by nitrogen gas. (2) Flow rate and heat input are increased gradually to the desired starting point, and the system is allowed to become steady as indicated by its pressure, temperature and flow rate. (3) Measurements of temperature, pressure, flow rate and heat input are taken and critical observations

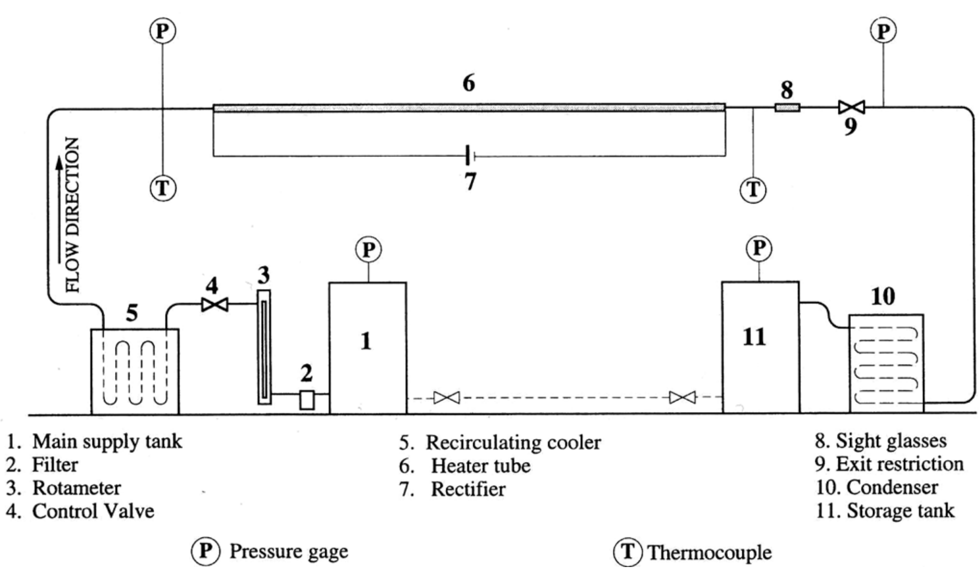

Fig. 5. Schematic diagram of the horizontal boiling system.

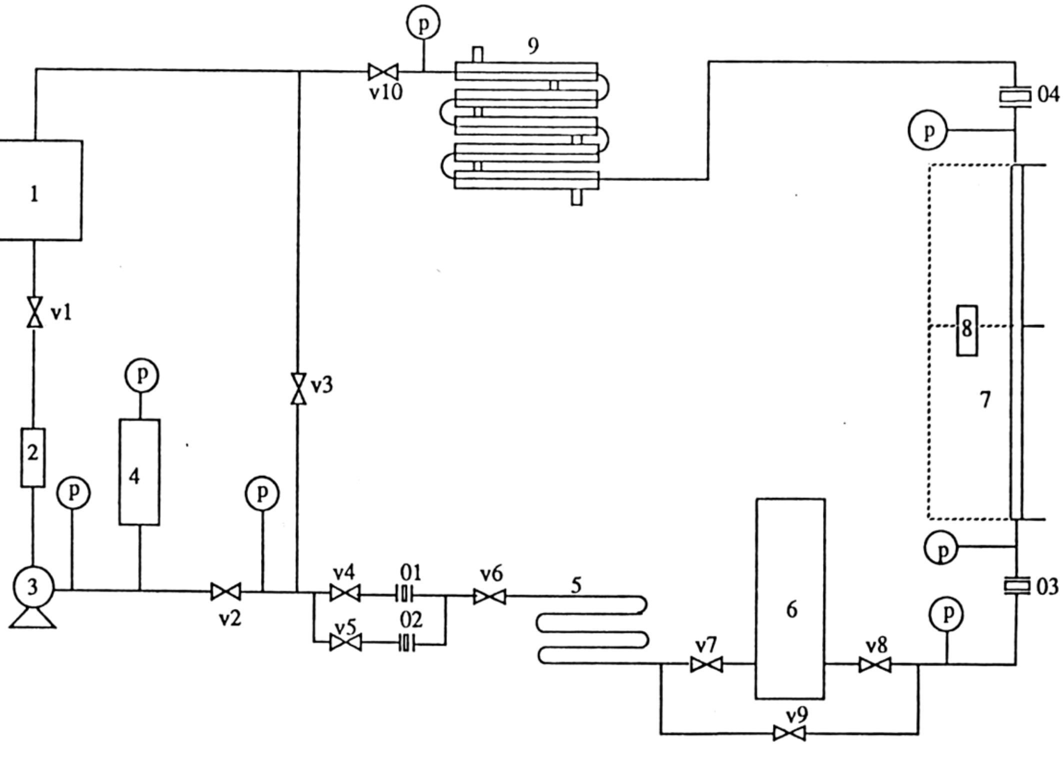

Fig. 6. Schematic diagram of high pressure boiling system.

are noted. (4) The mass flow rate is reduced by a small amount using the flow control valve. Following each adjustment the system is allowed to become steady after which step 3 is repeated until sustained oscillations are observed. The above procedure is repeated for different heat inputs and inlet temperatures. Different sets of experiments corresponding to different heat inputs were run for Freon-11 as the test fluid. Fig. 7 shows a typical plot of pressure-drop versus mass flow rate, pressure-drop being the pressure difference between surge tank and the system exit [88]. The flow rate and pressure recordings of the oscillations at the heater inlet are reproduced in Fig. 8. Fig. 9 shows the region where the high frequency oscillations (density-wave type) occur, and the typical density-wave type oscillations are shown in Fig. 10. The pressure-drop type oscillations are observed only for those flow conditions where the pressure-drop versus mass flow rate exhibits a negative slope, and the period of the oscillations are large. On the other hand, density-wave type oscillations appear on the positive slope of the pressure-drop versus mass flow rate curve. The period of the oscillations are on the order of a second.

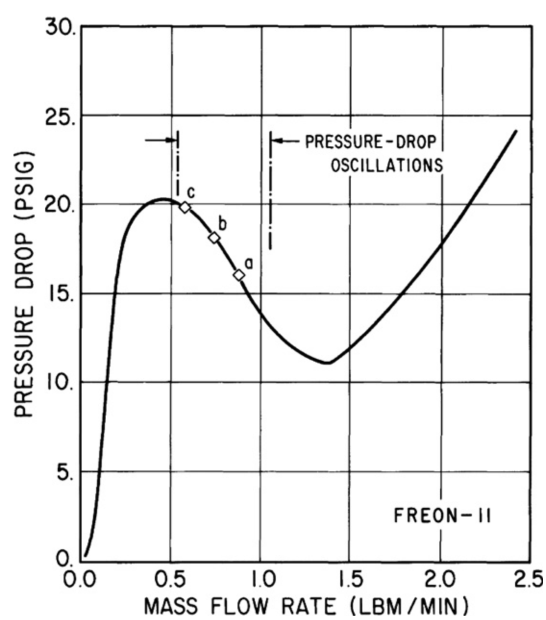

Fig. 7. Stability-instability boundary for pressure-drop type oscillations ($Q = 1023$ BTU/h, Ti = 25 °F).

In some of the experiments with film boiling, a completely different type of oscillation was observed [122]. In this type of oscillations the wall temperature fluctuated periodically between 262 °F and 396 °F (Fig. 11). These slow oscillations were called thermal oscillations because of the large fluctuations in wall temperature not encountered in other types. It appeared that a relatively rare combination of flow and heat transfer characteristics may be

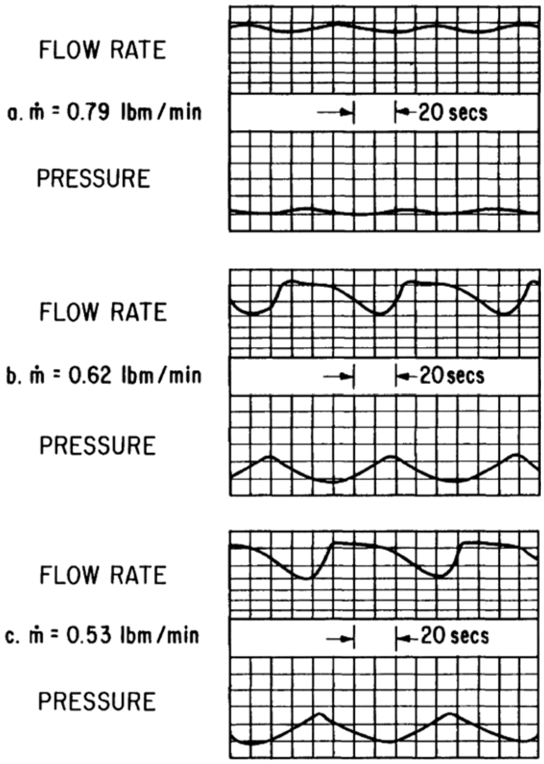

Fig. 8. Typical recordings of pressure-drop oscillations versus flow rate ($Q = 1023$ BTU/h, Ti = 25 °F).

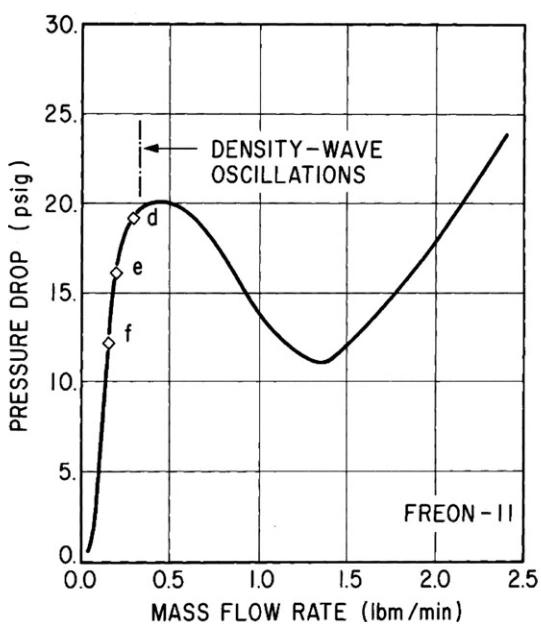

Fig. 9. Stability boundary for density-wave oscillations ($Q = 1023$ BTU/h, Ti = 25 °F).

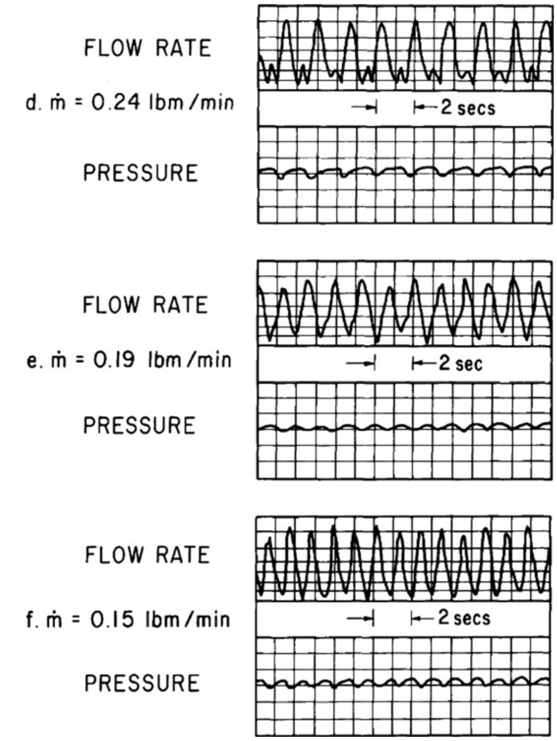

Fig. 10. Typical recordings of density-wave type oscillations.

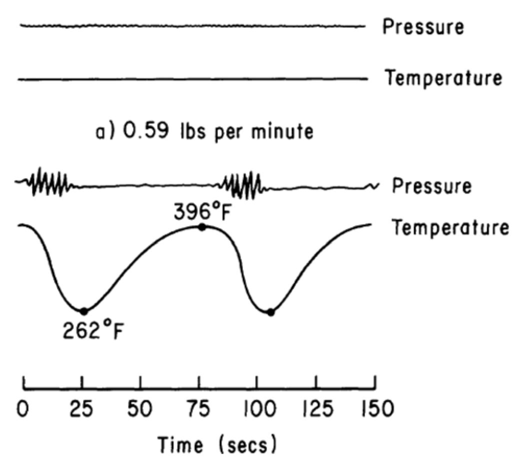

Fig. 11. Typical recordings of thermal oscillations. $Q = 700$ W, mass flow rate = 0.59 lbs/min.

necessary to permit thermal oscillations to occur [117]. The steady-state data were plotted as curves of the inlet plenum to system exit pressure-drop versus mass flow rate for various heater power inputs, and stable and unstable regions with respect to density-wave and pressure-drop type oscillations were obtained (Fig. 12). These are steady-state operating curves for the system. The region to the left of the black dots in the figure represents the region where the density-wave oscillations were encountered. As it can be seen from the figure, at sufficiently high flow rates where all the liquid flow persists throughout the system, the relation is roughly parabolic corresponding to turbulent liquid flow. As the flow rate is decreased and net vapor generation starts, a decrease in flow is accompanied by an increase in pressure-drop. The negative slope region widens and becomes steeper at increasingly higher heat inputs. A parametric study of the effects of mass flow rate, heat input and inlet liquid temperature on two-phase flow instabilities in a single upflow boiling channel has been conducted by Liu

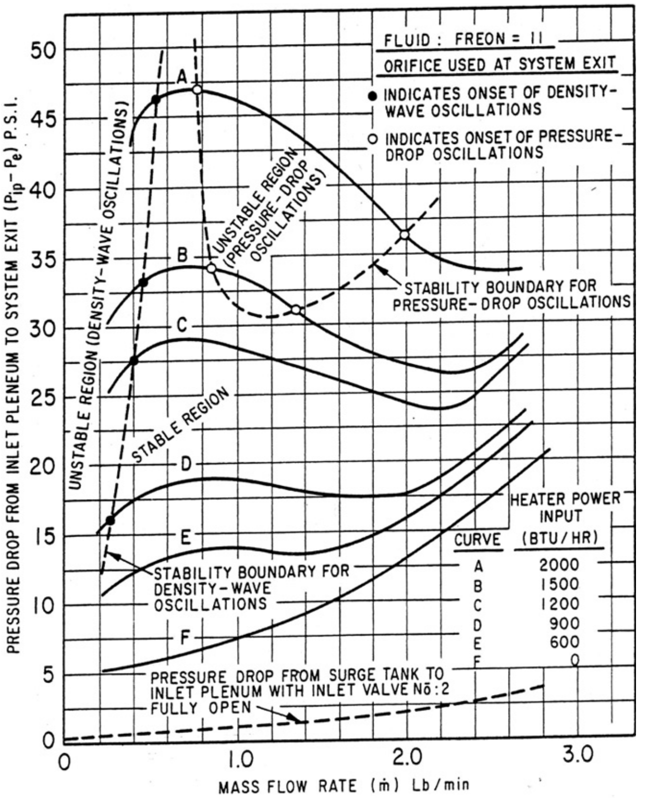

Fig. 12. Pressure drop from the inlet plenum to system exit versus mass flow rate steady-state relationships with exit orifice.

[12] and Liu and Kakac [33] and some of the results of density-wave type and pressure-drop type oscillations are reproduced in Table 2.

5.2. The effect of heat transfer augmentation on two-phase flow instabilities

The two-phase flow instabilities in an augmented single channel forced convection open loop up-flow system have been studied experimentally [111]. In the experimental investigation, the effects of difference heater surface configurations and inlet sub-cooling on two-phase flow instabilities were investigated. Freon-11 was used as the test fluid, and six different heater tubes with various inside surface configurations were tested at five different heat inputs. By the use of a subcooling controller, a wide range of subcooled fluid conditions have been obtained which enabled a detailed survey of the effects of inlet subcooling on the system stability [101,120]. A schematic diagram and the basic dimensions of the experimental setup are shown in Fig. 4. Six different tubes, which are shown in Fig. 13, were prepared and used during the experiments. The description of the tubes is in Table 3. The tubes are classified according to their effective diameters, which is defined as $D_e = (4V/\pi L)^{0.5}$, where V is the net inside volume and L is the length of the heater tube. Two basic types of oscillations, namely the pressure-drop type and the density-wave type, and var-

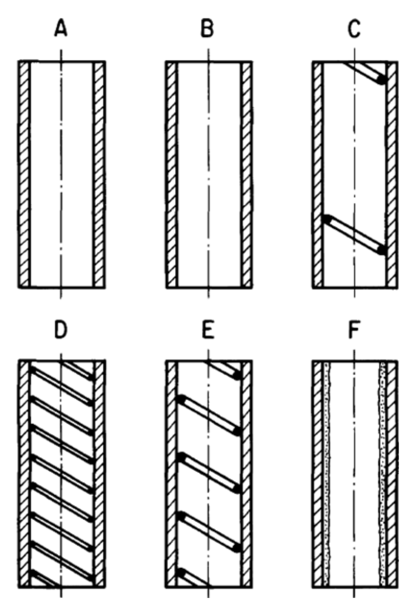

Fig. 13. Heater surface configurations for the vertical system.

Table 2
Experimental data of two-phase flow oscillations

Tube type	Q (W)	Ti (°C)	Mass flow rate (g/s)	Thermal oscillations		Pressure oscillations			
				Amplitude (°C)	Period (s)	Amplitude		Period	
						P.D. (bar)	D.W. (bar)	P.D. (s)	P.W. (s)
Coated	800	23	20.92	18.5	15	0.60	0.62	15	2.2
			16.28	28.8	21	0.75	0.61	21	2.5
			11.89	65.5	28	0.89	0.67	28	2.6
			9.06	100.2	40	0.94	0.50	40	2.7
			7.31	119.8	49	0.96	0.42	49	3.0
Bare	400	23	11.89	4.8	13	0.21	0.10	13	1.5
	600			10.4	18	0.52	0.22	18	1.5
	800			48.3	30	0.76	0.32	30	1.2
	1000			113.0	37	0.92	0.39	37	1.2
Coated	800	−10	7.31	162.0	68	1.19	0.51	68	3.0
		0		132.1	62	1.06	0.36	62	3.2
		10		124.8	50	0.98	0.43	50	3.2
		23		119.9	47	0.96	0.42	47	3.0

Table 3
Description of different heater tubes

Tube	Description of the tube	D_e (mm)
A	Bare	7.493
B	Threaded, 7.938 mm pitch	7.619
C	With internal spring of 0.794 mm wire diameter and 19.05 mm pitch	7.446
D	With internal spring of 0.432 mm wire diameter and 3.175 mm pitch	7.401
E	With internal spring of 1.191 mm wire diameter and 6.350 mm pitch	7.192
F	Inside surface coated with Union Carbide High Flux Coating	7.073

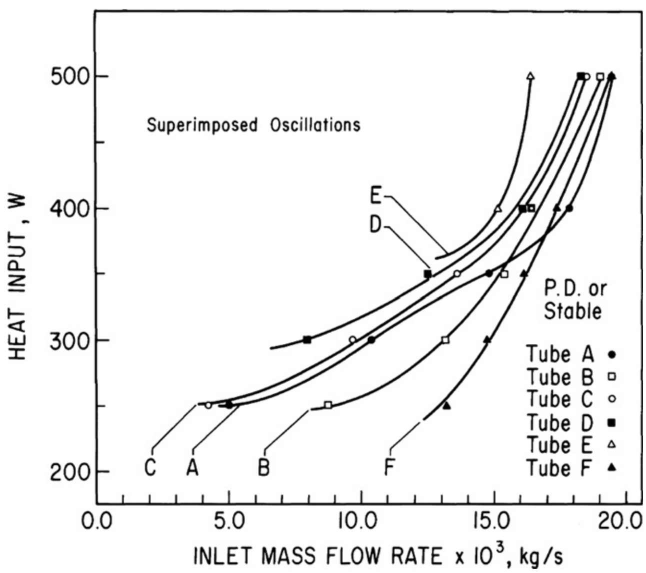

Fig. 14. Boundaries of the pressure-drop type oscillations with superimposed density-wave type oscillations in the vertical system.

ious forms of superposition of these oscillations were observed during the experiments. Boundaries for the superimposed oscillations for different tubes were obtained. It was observed that the curves for different heater tubes exhibited a similar pattern, with internally springed tubes having a narrower unstable region. The effect of surface configurations on heat transfer coefficient has also been examined. The results have been correlated for use in predicting the oscillating-state boiling heat transfer coefficient [104]. Comparing the bare tube with the other tubes, it has been found out that the time average local heat transfer coefficients during oscillations increased by 1–8% for different spring configurations, about 20% for the internally threaded tube, and up to 94% for the coated tube. From the investigation of Fig. 14, one can deduce that among the tested tubes, tubing with Linde High Flux Coating is the most unstable one [101].

5.3. Density-wave type oscillations

A high pressure water-vapor experimental set-up, as shown in Fig. 6, was built and used in order to investigate the basic mechanisms of different types of oscillations and effects of various parameters on these oscillations [64,65]. Demineralized and degasified water was fed to the test section (7) via surge tank (6) after being pressurized by a plunger pump (3). A by-pass controlled by (V3) was also installed in the system to ensure proper flow rate to the test section. Water flow rate to the test section was controlled by regulating valves (V4) and (V5). Water was heated to the desired inlet temperature in the pre-heater (5), and then heated again in the test section till boiling occurred. Two-phase mixture was sent back to water tank (1) through a chiller (9), where it was condensed back to below 50 °C. To eliminate the flow oscillation generated by the plunger pump, an absorber 4 was added to the system right after the pump. Steady-state characteristics of the system were first obtained and plotted on the pressure-drop versus mass flux plane. These characteristic curves were used to mark the instability boundaries. To study the effects of system pressure, mass flux, heat flux, inlet subcooling and the size of exit restriction on density-wave type oscillations, the following ranges were used to cover a broad two-phase flow region.

System pressure: 30–100 bar, mass flux: 600–1300 kg/m^2 s, inlet subcooling: 10–90 °C, exit restriction diameter ratios, β: 0.33, 0.417, 0.5, heat flux: 0–700 kW/m^2.

In this work, two-phase flow instabilities in boiling vertical and horizontal systems were presented. In the first phase of this study, the results of density-wave type oscillations in a single channel, forced convection boiling upflow system using water are given. A typical density-wave type oscillation is shown in Fig. 15. The limiting heat flux and

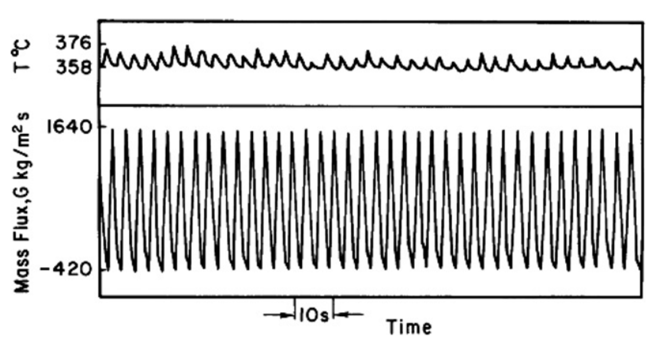

Fig. 15. Typical density-wave type oscillations ($p = 100$ bar, $G = 937.5$ kg/m^3 s, $T_{sub} = 90$ °C, $\beta = 0.33$, $Q = 580.5$ kW/m^2).

Table 4
The constants A and B in Eq. (6) as a function of exit restriction

β	A	B
0.33	0.2492	0.05902
0.42	0.2404	0.12020
0.50	0.2015	0.20370

quality of the onset of density-wave type oscillations were determined; their dependence on system pressure, mass flux, inlet subcooling and exit restriction was found and presented graphically. Also a simple correlation is formulated for the prediction of the limiting heat flux and quality of density-wave type oscillations: If one dimensionalizes the energy equation for homogenous two-phase flow, a dimensionless number N_Q can be obtained, Wang et al. [64,65].

$$N_Q = \frac{4Q_{s0}L_h}{h_l GD} \tag{4}$$

Replacing the heat flux of stable flows, by the limiting heat flux value, Q_c, and h_l by $(h_s - h_i)$, one can obtain:

$$N_Q = \frac{4Q_c L_h}{(h_s - h_i)GD} \tag{5}$$

N_Q can therefore be considered as a ratio relating the total heated length L_h to the length occupied by the single liquid phase in the test section. It also shows the relationship between limiting heat flux and fluid inlet subcooling [65].

$$x_C N_Q = A + B\frac{N_P}{N_{TS}} \tag{6}$$

where,

$$N_p = \frac{P_e \rho_L}{G^2 L_h}$$

$$N_{Ts} = \frac{h_s - h_o}{\Delta h_v}$$

Coefficients A and B are given in Table 4. In practice, the quality, x, in the system is to be found from thermal equilibrium conditions, and N_Q is to be calculated from Eq. (4). The product of x and N_Q is then compared with the value calculated from Eq. (6). If $xN_Q > x_C N_Q = A + BN_P/N_{TS}$, the system is in the region where density-wave type oscillations can occur. If $xN_Q < A + BN_P/N_{TS}$, the system will not experience this type of oscillation. Experimental data and those calculated from the above correlation are plotted in Fig. 16, which shows a clear linear relationship between them. This linear relationship suggests that the correlation (6) can be used satisfactorily to predict the threshold values of heat flux and quality of the density-wave type oscillations in a high pressure upflow system under the present experimental conditions.

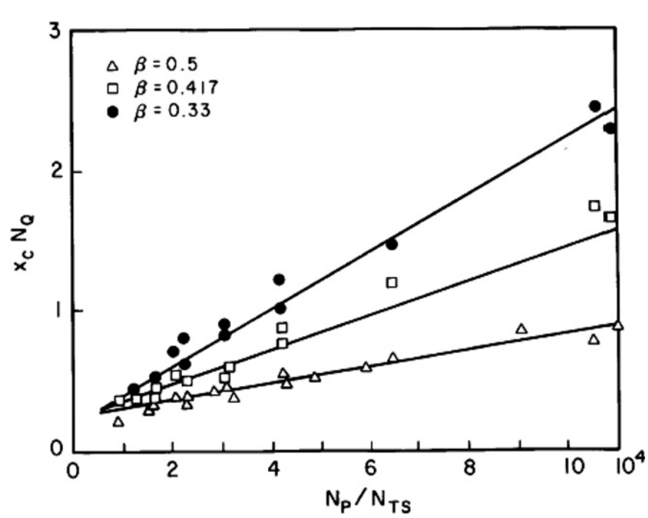

Fig. 16. Comparison of the correlation with experimental data.

6. Boiling flow instabilities in parallel channel systems

6.1. Two parallel channel system

In a two parallel boiling upflow system – two parallel channels, containing a heater each, installed between an inlet and exit plenum and was fed with test fluid from a pressurized tank was studied [49]. The experimental setup was the same as described in Fig. 4. Two heater geometries used in these experiments are shown in Fig. 17a and b. Freon-11 was used as the test fluid, and various experiments were carried out. In each set of tests, the inlet temperature of the test liquid and power inputs to the heater were fixed. The flow rate through the system was then varied over the range allowed by system characteristics. In general, the system would be stable within the portions of the flow range and unstable in other portions. In the unstable region, two different modes of oscillations were observed. They were the same as those observed in single channel upward boiling flows. These oscillations did not produce any noticeable temperature changes in the heater wall temperatures. The steady-state experimental data have been plotted as curves of the surge tank to system exit pressure-drop versus total mass flow rate for various heat input combinations. Fig. 18 shows the steady-state characteristics and stable–unstable regions of the system for different heat inputs and Fig. 19 shows recordings of pure density-wave type oscillations. A study of the figure with regard to the density-wave type oscillations indicates that: (1) increase in total flow rate increases stability; (2) increase in pressure-drop increases stability; and (3) increase in difference between the heat input into individual channels decrease stability. A study of the figure with regard to

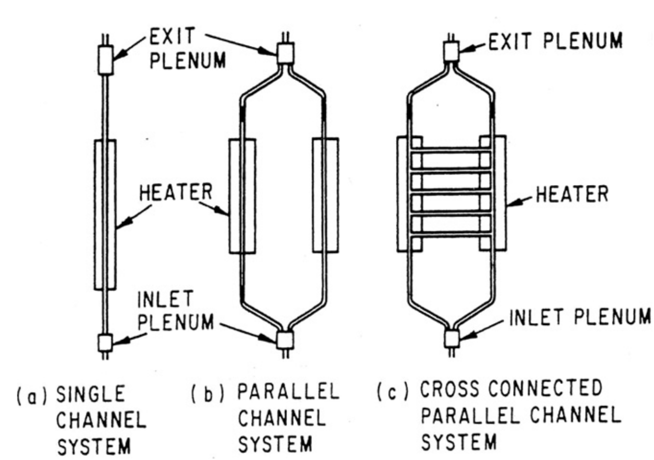

Fig. 17. Schematic diagrams of various heater geometries.

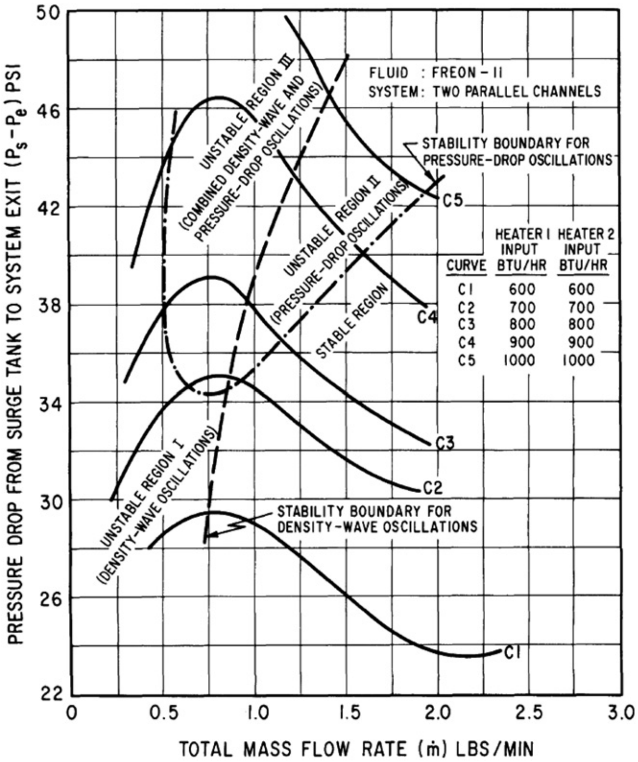

Fig. 18. Pressure drop from surge tank to system exit versus total mass flow rate steady-state relationship for same heat inputs for two parallel channels.

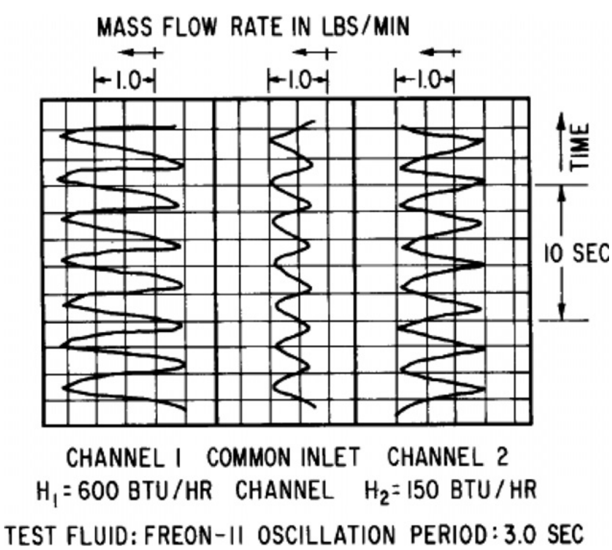

Fig. 19. Recordings of pure density-wave type oscillations.

the pressure-drop type oscillations, indicates that (1) increase in total flow rate increases stability; (2) increase in system pressure-drop decreases stability; (3) increase in total heat input decreases stability.

6.2. Cross-connected parallel channel upflow system

The parallel channel systems such as the one discussed in the last section cannot represent all the boiling water reactors from the point of view of fluid dynamics. In many reactor heat removal systems, there are cross-connections between the parallel vertical coolant channels (Fig. 17c). This system was found to be more stable than the others [55,56]. Experimental apparatus and procedures were similar to the one used in the experiments discussed in the previous sections. Density-wave type oscillations occurred in the positive slope regions of the boiling flow pressure-drop versus flow rate curves. Whenever density-wave type (flow and pressure) oscillations started, they would be present in all the channels of the system. The oscillations in the two parallel vertical channels were in phase. The experimental data have been plotted as curves of the surge tank to system exit pressure-drop versus total mass flow rate for various heat input combinations. Fig. 20 shows the family of curves for equal heat inputs into heater 1 and 2. A study of the figure, with regard to the oscillations indicates that: increases in total flow rate increases stability; increases in system pressure-drop decrease stability; increase in heat input decreases stability; and equal heat inputs make the system more stable [51]. The pressure-drop type oscillations in the two parallel vertical channels were in phase; and the oscillations in the common inlet channels represented the algebraic sum of the oscillations in parallel channels. Their amplitudes were relatively large as compared to the pressure-drop oscillation amplitudes in the parallel channels. The pressure-drop oscillations could be stopped by introducing either pressure-drops at the inlets of the two heaters

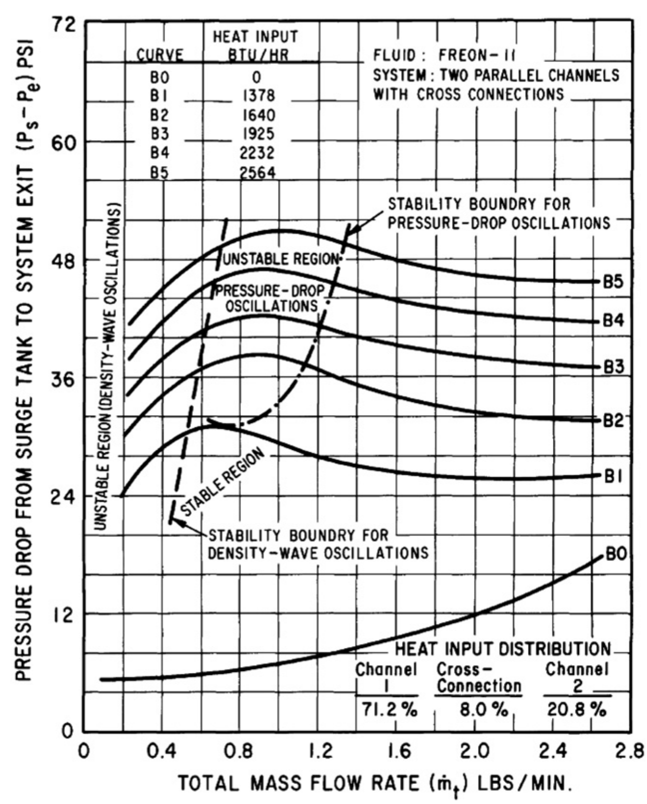

Fig. 20. Pressure drop from surge tank to system exit versus total flow rate relationship for equal heat inputs with cross-connections.

or a pressure-drop between the inlet plenum and the surge tank; the latter was found more effective.

6.3. Transient boiling flow instabilities in a multi-channel upflow system

In this study, the transient boiling flow instabilities in a four parallel channel upflow system with and without cross-connections were experimentally investigated using Freon-11 as the test fluid (Fig. 21). Several series of transient experiments were run using a step increase in power input with a constant flow control valve setting as well as step decrease in flow rate with a constant heat input. Experiments were conducted with equal and unequal heat inputs and the results were compared with one another. A comparison of the transient instabilities observed in four parallel channel systems with and without cross connections was also made [55,56]. The experiments performed for investigating the transient two-phase flow instabilities in four-parallel channel upflow systems can be separated mainly into three groups: (1) the experiments on cross-connected four parallel channel system with equal heating of channels; (2) the experiments on four parallel channels system without cross-connections with equal heating of channels; (3) the experiments on four parallel channel system without cross-connections with unequal heating of channels. In each of these group, input to the channels was held constant while the flow rate was suddenly reduced by a certain amount by partially closing the control valve and this was repeated for different heat inputs. In the second series of experiments, the setting of the flow control valve was not changed while the heat input to the channels was incrementally increased. Fig. 22 is a typical plot of the test section pressure-drop versus total mass flow rate for various constant heat inputs and incrementally decreasing flow rates, for the cross-connected system. It was found that the cross-connected four-parallel-channel system is more stable than the four-parallel-channel system without cross-connections, with regard to transient pressure-drop and density-wave type oscillations. Also the minimum heat input at which this system can operate without any transient pressure-drop type oscillations is higher in the cross-connected system than that of the system without cross-connections. The periods of the transient pressure-drop type oscillations in the system without cross-connections are relatively larger than those of the system with cross-connections. The periods of the transient density-wave type oscillations are not much different in these two systems. This is due to the fact that, not the heater geometry, the exit restriction is the determining factor. In both of the systems, the transient density-wave and pressure-drop type oscillations are in phase in all of the channels. With regard to the transient and sustained pressure-drop type oscilla-

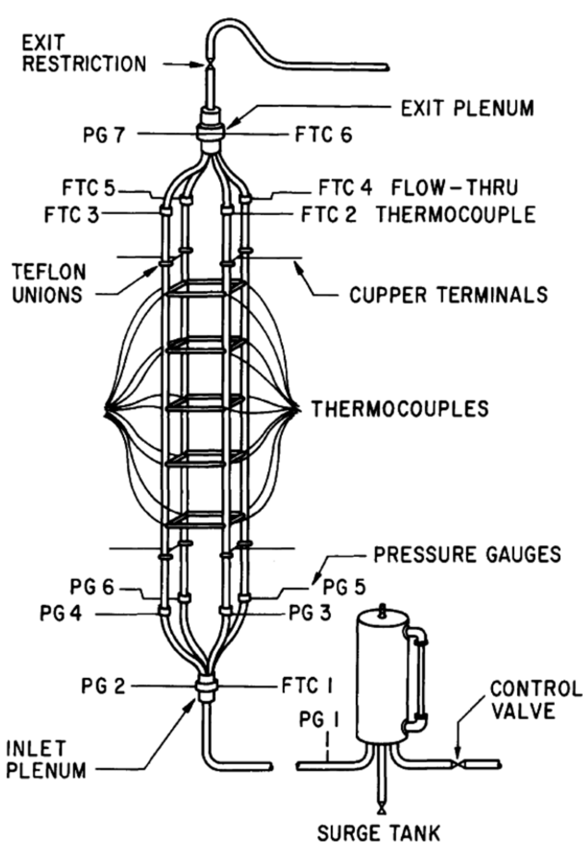

Fig. 21. Schematic diagram of test section of four channels with cross-connections.

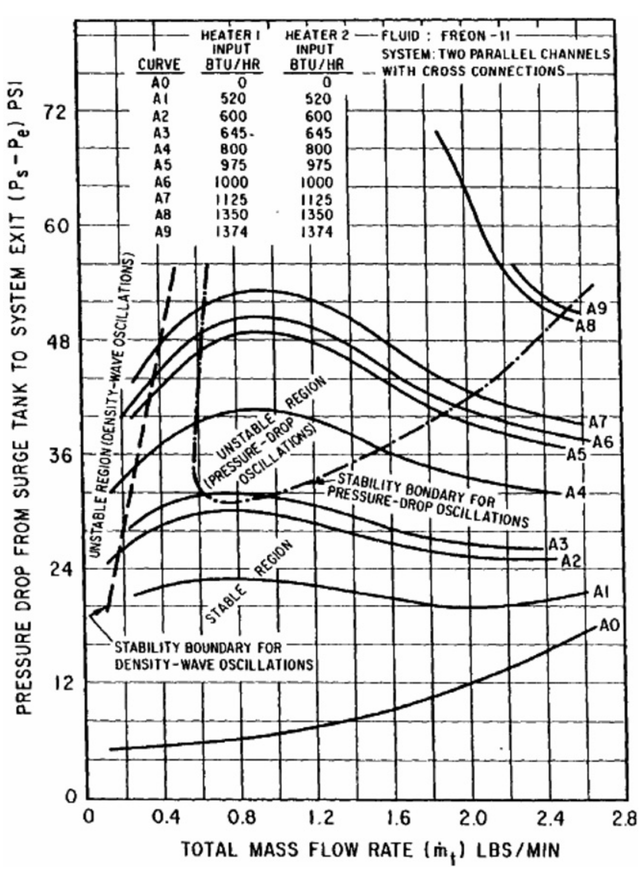

Fig. 22. Pressure drop from surge tank to system exit versus total mass flow rate relationship for unequal heat inputs with cross-connections.

tions, the equally heated system is more stable than the unequally heated system. The difference between the two systems is not large, even though they show similar transient pressure-drop type stability behavior, if the heat input per channel rather than total heat input is taken into consideration.

6.4. Sustained instabilities in a cross-connected four parallel channel upflow system

In this experimental investigation, the instabilities induced by forced convection upflow boiling in a four parallel channel system with cross-flow mixing have been studied using Freon-11 as the test fluid. A comparison of the instabilities of the present system with those of other simpler systems has been made with regard to the phase relationships, the periods of oscillation, and the stability. The present system was found to be more stable than the others [55]. The experimental apparatus was the same as it is shown in Fig. 4. The test section is shown in Fig. 21. It has been observed that whenever the density-wave type and pressure-drop type oscillations occurred, they were present in all channels of the system. The oscillations in the four parallel channels were in phase. Fig. 23 shows typical total pressure drop between the surge tank and the exit as a function of the total mass flow rate for various heat inputs. Multi-channel boiling system has been studied by Ozawa et al. [123] and Nakanishi et al. [124]. They found that if the flow was distributed uniformly among all channels, they would oscillate with the same amplitudes and equal phase lag. Akagaw et al. [38] studied three parallel channel system. It is reported that two channels oscillated in phase with different amplitudes and the third channel oscillated 180° out of phase with the former. Darcy [125,126] investigated the boiling flow instabilities in three parallel internally heated annular channels. He observed three modes of oscillations: (1) two channels oscillated with the same amplitude and in phase and the third oscillated 180° out of phase; (2) two channels oscillated with approximately the same amplitude and 180° out of phase and the third channel did not oscillate; and (3) all three channels oscillated in a kind of three-phase type of oscillation.

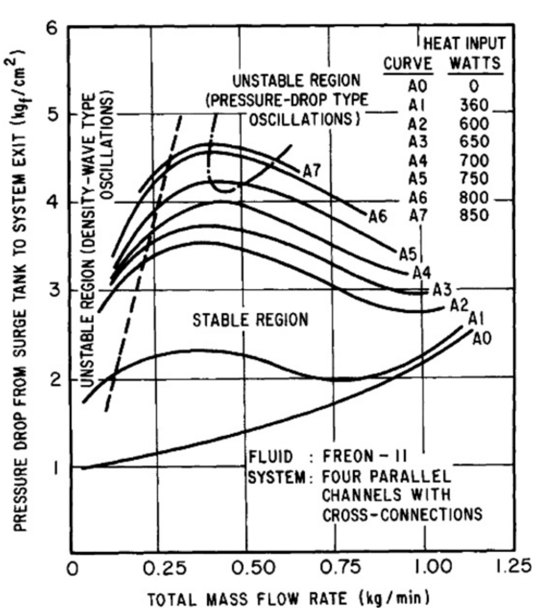

Fig. 23. Pressure drop from surge tank to system exit versus total mass flow rate steady-state relationship.

6.5. Comparison of various channel systems

In order to compare the results of the various channel system geometries discussed, their stability boundaries have been plotted in a total heat input versus total mass flow rate plane as shown in Fig. 24 [50,51,88,122]. In the figure, stability boundaries shown are those which occur first as the flow rate is reduced. The region to the left of the curve is unstable region, and region to the right is stable. A study of the figure shows that for equal heat input, the three systems, namely single channel, two parallel channel and cross-connected two parallel channel (curve A, B, and E) behave in almost the same manner, especially for the pressure-drop instability boundaries. With regard to density-wave type oscillations, the most stable system is the cross-connected four parallel channel system with equal heat input into the parallel channels (curve G), followed by the single channel system (curve A), the cross-connected parallel system with unequal heat inputs (curve F), the parallel channel system with equal heat inputs (curve B), the

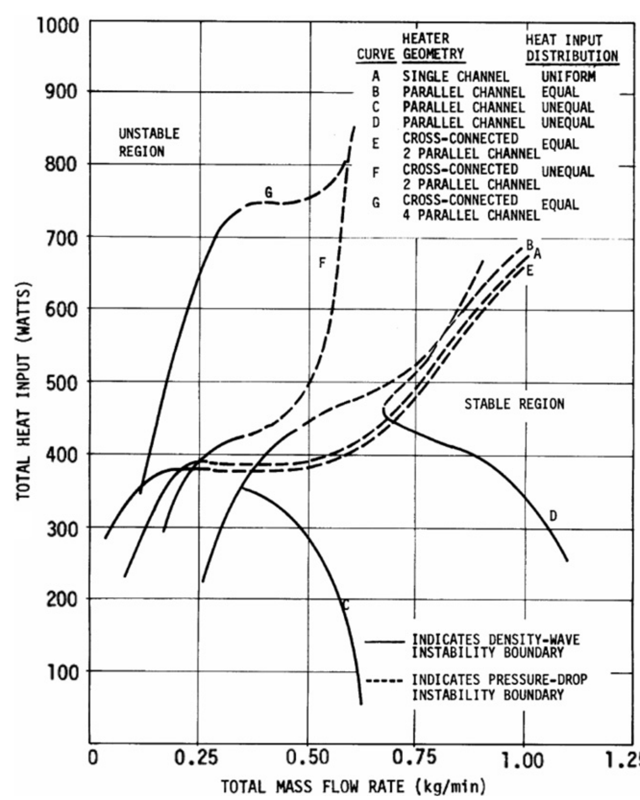

Fig. 24. Stability boundary comparison of various channel systems.

parallel channel system with a constant heat input of 600 BTU/h, to channel 1 (curve C) and the parallel channel system with a constant heat input of 900 BTU/h, to channel 1 (curve D), in that order. The last two systems look very unstable in the figure as compared with the other systems. This is caused by the fact that, while the flow rate in channel 1 in each case was low enough for the density-wave type oscillations to occur, the flow rate in channel 2 was high. Consequently, the total flow rate in each case was biased towards higher flow rates. With regard to the pressure-drop type oscillations, the most stable system is the cross-connected four parallel system with equal heat inputs (curve G) followed by the cross-connected two parallel with unequal heat inputs (curve F), the parallel channel system with a constant heat input of 900 BTU/h. to channel 1 (curve D), the parallel channel system (curve A) and the cross-connected parallel channel systems with equal heat inputs (curve E), in that order.

7. Mathematical modeling of instabilities and experimentation

In general, modeling two-phase phenomena starts with a formulation of the conservation equations. Boundary conditions are then defined for particular problems under consideration. At each step of the formulation and solution, various physical assumptions and approximations need to be made, which have to be justified later by the experimental conditions and results.

Different models are used to describe two-phase flow behavior in channels. In the most general formulation of the two-phase flow problem, the conservation equations are written separately for each of the phases. This is called the separated-flow model [127]. The resulting equations form the basis of the separated-flow model, or so-called unequal-velocity and unequal temperature (UVUT) model. In most of the practical problems, the one-dimensional time dependent equations are used, and the problem of the correct statistical and cross-sectional averaging in flow passages arises. Various forms of the conservation equations have been given [128,129,133,130–132]. The model can be written using either time averages [133] or space averages [134]. If it is written using space averages, the two-phase model can only deal with transient flow problems with a single space variable. To solve the set of six conservation equations written for each phase, seven consecutive laws are required, viz. four at the wall (friction and heat transfer for the two phases) and three at the interface of the phases (mass, momentum and energy transfer). Although the separated-flow model seems to be the most satisfactory in theory, it has been found complicated to be used in problems of practical importance because of the knowledge of the seven consecutive laws that is required. Various analyses accomplished so far can be collected under two general headings, namely the homogenous flow model and slip-flow model. In most of the studies of two-phase flow instabilities, the *homogenous equilibrium model* is widely used. At the onset, the homogenous model is for the prediction of the steady-state behavior and two-phase oscillations. This model treats the two-phase flow as the flow of single phase compressible fluid. The velocity of both phases is assumed to be equal, and the temperature is taken to be the saturated temperature. These assumptions are valid for rapid interfacial rates of heat and momentum transfer. Therefore, the model can be expected to be most applicable for those two-phase flow regimes where the phases are well-mixed, such as bubbly, churn, or mist flow regimes.

The slip-flow model considers the phases to be artificially segregated into two streams—one of liquid and one of vapor. In this model's simplest form, the phases are in thermodynamic equilibrium, and each phase is assumed to travel at a mean velocity. In the more general case of unequal phase velocities, an interphase constitutive relationship is needed. This can be in the form of a slip correlation relating the slip ratio, $S = u_g/u_f$, to the quality and pressure, to a drift velocity.

The drift-flux model of Zuber and Findlay [117] is relatively simple to use, and realistic enough for most purposes. In Wallis' (1969) drift-flux formulation, the void fraction and volumetric flux are assumed to be uniformly distributed over the channel cross-section. Flow regime effects are not accounted for in the Wallis' model. In the different formulations [135], the drift-flux model is an approximation of the more rigorous two-fluid models. It is presumably most valid for cases in which the drift velocity is significant compared with the volumetric flux. This limits its usefulness to the bubbly, slug and churn flow patterns. The models enable the researchers to predict the influence of the system geometry, pressure, temperature, quality, inlet and exit restrictions, and the mass flow rate, as well as effects of property variations on the system behavior. The two-phase flow oscillations can be simulated, and the amplitudes and periods of the sustained oscillations can be predicted. The stability boundaries for various operating conditions can also be obtained. The fundamentals of these two models used by various researchers will be explained in the following sections.

7.1. Drift-flux model

The drift-flux formulation, which has gained much acclaim in the last decade, takes the relative velocity between the phases into account, while assuming thermodynamic equilibrium. The basic assumptions used in the drift-flux model were outlined in [136,137]. With the assumptions outlined, the generalized one-dimensional transient governing equations of two-phase flow were reduced to the following:

Continuity:

$$\frac{\partial}{\partial t}[\rho_l(1-\psi)+\rho_v\psi]+\frac{\partial}{\partial z}[\rho_l u_l(1-\psi)+\rho_v u_v\psi]=0 \tag{7}$$

Energy:

$$\frac{\partial}{\partial t}[\rho_l h_l(1-\psi)+\rho_v h_v \psi]+\frac{\partial}{\partial z}[\rho_l u_l h_l(1-\psi)+\rho_v u_v h_v \psi]=\phi \tag{8}$$

Momentum:

$$-\frac{\partial p}{\partial z}=\frac{\partial}{\partial t}[\rho_l u_l(1-\psi)+\rho_v u_v \psi]+\left(\frac{\partial p}{\partial z}\right)_{\text{fric}} + \frac{\partial}{\partial z}\left(G^2\left[\frac{(1-x)^2}{\rho_l(1-\psi)}+\frac{x^2}{\rho_v \psi}\right]\right)+g[\rho_l(1-\psi)+\rho_v \psi] \tag{9}$$

where the subscripts 'l' and 'v' refer to liquid and vapor phases, respectively. The kinetic and potential energy terms have been neglected.

7.2. Homogenous model

The one-dimensional transient conservative equations, which are used to describe the two-phase flow inside a channel are based on the homogenous equilibrium model.

Akyuzlu et al. [88] and Dogan et al. [138] solved the following conservation equations for mass, momentum and energy together with the equation of state for unsteady homogenous viscous flow.

$$\frac{\partial \rho}{\partial t}=-u\frac{\partial \rho}{\partial z}-\rho\frac{\partial u}{\partial z} \tag{10}$$

$$\rho\frac{\partial u}{\partial t}=-\rho u\frac{\partial u}{\partial z}-\frac{\partial P}{\partial z}-2\frac{f}{D}\rho u^2-\rho g \tag{11}$$

$$\frac{\partial h}{\partial t}=-u\frac{\partial h}{\partial z}+\frac{1}{\rho}\Phi \tag{12}$$

where Φ is the heat input per unit volume of fluid and f stands for the two-phase friction factor. The properties like enthalpy and density are the mixture fluid properties. The steady-state flow characteristics are obtained for various heat inputs and inlet temperatures by solving the conservation equations together with the equation of state by using an implicit finite-difference technique. In the analysis of low frequency oscillations it is assumed that the quasi-steady-state conditions prevail in the heater. The system equations obtained with this assumption are solved under constant exit pressure and constant container pressure boundary conditions using the finite-difference technique. Two methods of approach are adapted in solving the non-linear hyperbolic equations which describe the systems at low mass flow rates where the density-wave type oscillations are observed. They are the explicit integral momentum method (EIM) and the explicit finite-difference method (EFD). A comparison of the results with experiments and other mathematical models is discussed. By the use of a computer program developed, the steady-state system pressure-drop for constant heat flux and for various inlet temperatures are obtained for various mass flow rates (Fig. 25). Sample plots of the numerical simulation of pressure-drop and density-wave type inlet velocity oscillations at 1023 BTU/h heat input and 25 °F inlet temperature are presented in Figs. 26 and 27, respectively.

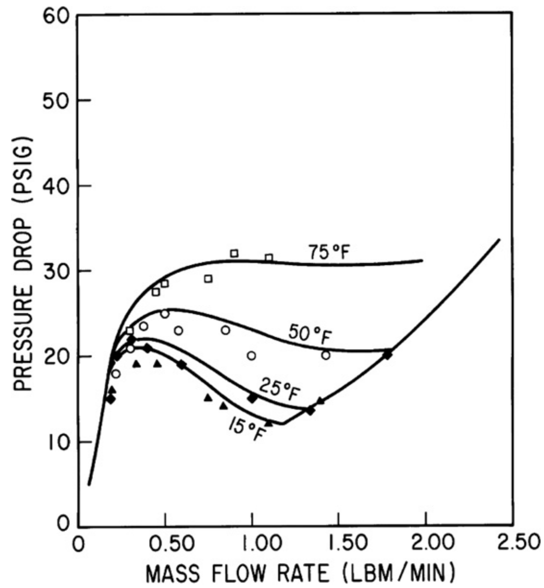

Fig. 25. Comparison of theoretical and experimental steady-state flow characteristics for various inlet temperatures ($Q = 1023$ BTU/h).

Dogan et al. [138] developed a numerical model to predict the steady-state and transient behavior of forced convection boiling two-phase flow in a vertical single channel. The model is based on the assumption of homogenous two-phase flow and thermodynamic equilibrium of the phases.

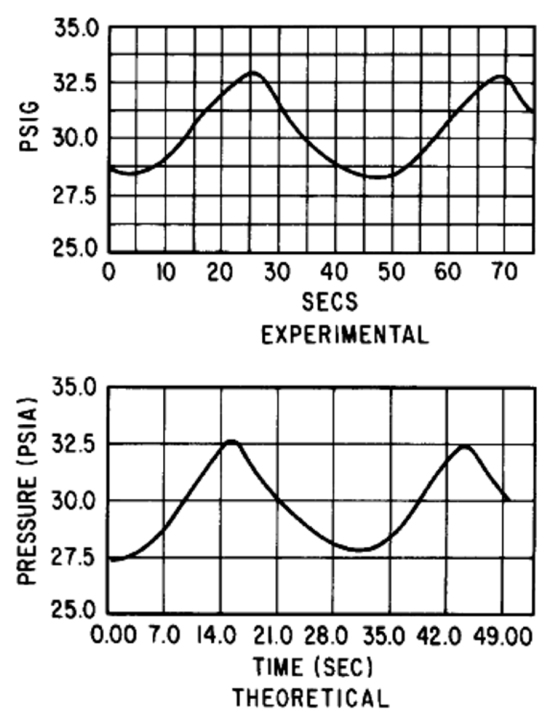

Fig. 26. Numerical simulation of pressure-drop oscillations.

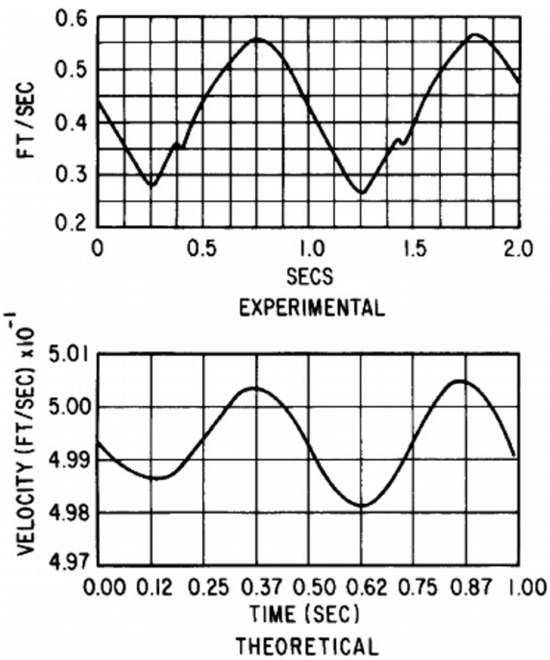

Fig. 27. Numerical simulation of density-wave type oscillations.

The model is used to study the effects of heat input, inlet subcooling and flow rate on the system behavior. For comparison purposes, an experimental investigation was conducted using a single channel, electrically heated, forced-convection upflow system. Steady-state operating characteristics, and stable and unstable regions, are determined as a function of heat flux, inlet subcooling and mass flow rate. The conditions provided by the steady-state solution are used as the initial conditions for the time dependent solution. The boundary conditions are constant exit pressure and constant inlet flow rate to the surge tank. The homogenous two-phase flow model findings were verified by the experimental results and satisfactory agreement was obtained.

7.3. Thermal oscillations

In general, modeling of two-phase phenomena starts with the formulation of the conservation equations. Boundary conditions are then defined for the particular problem under consideration. At each step of the formulation and solution, various physical assumptions and approximations need to be made, which have to be justified later by the experimental conditions and results. In some studies for the prediction of the steady-state behavior in two-phase oscillations, a homogenous model is used. This model treats the two-phase flow as the flow of a single-phase compressible fluid. The velocities of both phases are assumed to be equal, and the temperature is taken to be the saturated temperature. These assumptions are valid for rapid interfacial rates of heat and momentum transfer. The model predicted the influences of the system geometry, pressure, temperature, quality, inlet and exit restrictions, and the mass flow rate, as well as effects of property variation on the system behavior. The one-dimensional transient conservation equations based on the homogenous equilibrium model are similar to those for compressible single-phase flow, although the constitutive relations are quite different. These relations, that is, the two-phase friction and heat transfer correlations, along with the density changes, account for two-phase phenomena. Under these assumptions and definitions, the conservation equations for mass, momentum and energy, as applied to one-dimensional, unsteady, homogenous, viscous flow, can be written as Eqs. (10)–(12). These equations have been written for a uniform flow area with diameter d. The kinetic and potential energy terms have been neglected. In addition to the conservation equations, equations of state are needed, which can be written as follows [117,138,139]:

For subcooled liquid properties:

$$\zeta_l = \zeta_l(T) \tag{13}$$

where ζ represents a general thermodynamic property which has been correlated in a polynomial form from the available data in the literature.

For saturated liquid and vapor properties:

$$\zeta_l = \zeta_l(P) \tag{14}$$

$$\zeta_v = \zeta_v(P) \tag{15}$$

In two-phase regime:

Quality:

$$x = \frac{(h - h_{sat})}{h_{lv}} \tag{16}$$

Density:

$$\rho = \frac{\rho_l}{\left[1 + x\left(\frac{\rho_l}{\rho_v} - 1\right)\right]} \tag{17}$$

Viscosity:

$$\mu = (1 - x)\mu_l + x\mu_v \tag{18}$$

where the subscripts 'l' and 'v' refer to liquid and vapor phases, respectively.

The two-phase flow friction coefficient was obtained experimentally [100].

$$f = 0.16(\rho u d/\mu_l)^{-0.25} \tag{19}$$

The heat input into the fluid, Q_L, is given by:

$$Q_L = \alpha A_h (T_w - T_f) \tag{20}$$

where α is the heat transfer coefficient for boiling flow, A_h is the heater inner surface area, T_w the heater inside wall temperature, and T_f is an appropriately averaged fluid temperature. The expression for α is usually very complicated. The complication arises mainly from the existence of boiling, which may assume different modes depending on the heat surface, magnitude of the heat flux, and the flow conditions. Under the conditions of the present experimental study, the two-phase heat transfer coefficient was obtained experimentally and was plotted as a function of quality. The following correlation was obtained to calculate the heat transfer coefficient in two-phase:

$$\alpha_{lv} = 1.2\alpha_l - 0.2\alpha_l\left(\frac{0.3 - x}{x}\right)^n; \quad x \leqslant 0.3 \tag{21}$$

$$\alpha_{lv} = 1.2\alpha_l - 0.2\alpha_l\left(\frac{x - 0.3}{x}\right)^n; \quad 0.3 \leqslant x \leqslant 10 \tag{22}$$

where

$$n = \frac{\ln\left(\frac{1.2\alpha_l - \alpha_v}{0.2\alpha_l}\right)}{0.8473} \tag{23}$$

The single-phase heat transfer coefficient α_l (liquid) and α_v (vapor) in Eqs. (22) and (26) are calculated from Gnielinski's correlations for $Re_p > 2300$ [140]:

$$Nu_p = \frac{\alpha_p d}{k_p} = \frac{(f_p/2)(Re_p - 1000)Pr_p}{\left[1 + 12.7(f_p/2)^{0.5}(Pr_p^{2/3} - 1)\right]},$$
$$p = l, v \tag{24}$$

where

$$f_p = [1.58\ln(Re_p) - 3.28]^{-2} \tag{25}$$

Experimental values of the two-phase heat transfer coefficient can also be correlated by the following simple correlation under the present experimental conditions:

$$\alpha_{lv} = 1.2\alpha_l - (0.3 - x)^2[2.222\alpha_l + x(0.227\alpha_l - 2.041\alpha_v)] \tag{26}$$

The uncertainty associated with the correlation in Eqs. (21)–(23) was about ±15%.

7.3.1. Steady-state characteristics

The study of two-phase flow dynamic instabilities, in general, requires steady-state characteristics of the flow for various states of the working fluid (liquid, liquid + vapor, vapor). These characteristics, which are the steady-state solutions of the conservation equations, are also used to determine the initial conditions for the various types of oscillations. Therefore, the steady-state solutions are obtained for various inputs to the heater channel for the range of interest of fluid mass flow rates. The single channel upward boiling flow apparatus, systematically sketched in Fig. 4, is simulated for this model as shown in Fig. 28. Freon-11 enters the system from the main tank at the point designated as "I." The outlet of the surge tank is "O." The part of the system from the surge tank to the inlet of the heater is called Region 2. The subcooled part of the heater is Region 3, and the result of the heater where boiling takes place is called Region 4. Region 5 extends from the exit.

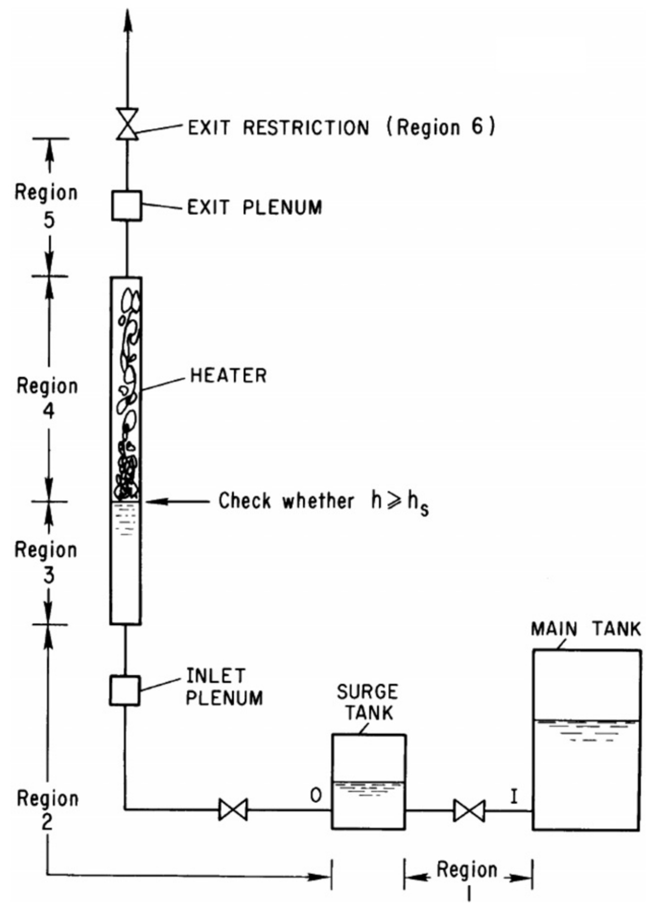

Fig. 28. Schematic drawing of the mathematical model.

The conservation equations for steady-state reduce to:

$$\frac{\partial u}{\partial z} = 0 \tag{27}$$

$$\frac{\partial P}{\partial z} = -2\frac{f}{D}\rho u^2 - \rho g \tag{28}$$

$$\frac{\partial h}{\partial z} = 0 \tag{29}$$

Region 3. The single-phase liquid in this region is assumed to be incompressible; therefore, the continuity and momentum equations are given by Eqs. (27) and (28), respectively. The heat input into the system is specified in this region:

$$\Phi = \text{constant} \tag{30}$$

Thus, the energy equation for steady-state becomes

$$\rho u\frac{\partial h}{\partial z} = \Phi \tag{31}$$

Region 4. The conservation equations for this region are given by:

$$\frac{\partial(\rho u)}{\partial z} = 0 \tag{32}$$

Momentum:

$$\rho u\frac{\partial u}{\partial z} + \frac{\partial P}{\partial z} = -2\frac{f}{D}\rho u^2 - \rho g \tag{33}$$

Energy:

$$\rho u \frac{\partial h}{\partial z} = \Phi \tag{34}$$

Regions 5 and 6. Continuity and momentum equations are given by Eqs. 32 and 33. Since there is no heat input in this region, the energy equation reduces to Eq. (34).

7.3.2. Exit restriction

The flow restriction at the system exit is a sharp-edged orifice with an orifice diameter of 0.0625”. An empirical correlation, based on previous data, is used to calculate the pressure drop across the restriction. The liquid-phase pressure-drop across the exit restriction is correlated as:

$$\Delta P_e = 635(G^2/\rho_l) \tag{35}$$

where G is the mass velocity.

To correlate the two-phase pressure-drop data, a two-phase friction multiplier model is used. The two-phase friction multiplier F_m has been correlated as a function of the exit quality, x_e, as:

$$F_m = 1 + 46.73x_e + 203.49x_e^2 - 195.32x_e^3 + 97.72x_e^4 \tag{36}$$

Thus the two-phase pressure-drop across the exit restriction becomes:

$$\Delta P_e = F_m 635(G^2/\rho_l) \tag{37}$$

7.3.3. Boundary conditions

The conservation equations together with the equations of state and the constitutive relations are to be solved for the following boundary conditions: Constant inlet temperature, $T_i = \text{constant}$, constant heat input, $Q_I = \text{constant}$, and constant exit pressure, $P_e = \text{constant}$.

7.3.4. Method of solution

The flow channel for the systems, as shown in Fig. 28, is divided into n segments in the z direction, parallel to the fluid flow. The numerical solution is obtained by the finite differences technique. During pressure-drop type oscillations, the mass flow rate, heat transfer coefficient, and heat input into the fluid keep changing. However, the heat generated in the heater wall is constant. Therefore, when the limit cycle enters the liquid region, the wall temperature decreases whereas the liquid heat transfer coefficient is usually high. When the limit cycle enters the vapor region, the wall temperature increases. Thus, the wall temperature keeps fluctuating during the limit cycle. These are called thermal oscillations.

The rate of heat transfer into the fluid is given by:

$$Q_{I,i} = \pi d \Delta z \alpha_i (T_{w,i} - T_{f,i}) \tag{38}$$

The heater wall temperature can be calculated from the energy balance for the heater, yielding:

$$\frac{d(T_w)}{dt} = \frac{(Q_o - Q_I)}{m_h c_h} \tag{39}$$

In the pressure-drop type oscillations model, fluid parameters and properties are calculated along the system during the oscillations.

7.3.5. Model for pressure-drop type oscillations

Pressure-drop-type oscillations occur in systems that have a compressible volume upstream of the heated section and are triggered by a static instability. They are usually confined to the middle portion of the negative slope region of the pressure-drop versus mass flow rate curve. The periods of the typical pressure-drop oscillations are much larger than the residence time of any fluid particle in the flow; hence, the pressure-drop-type oscillations are assumed to take place as a succession of quasi-steady-state operating points of the system. The conditions provided by the steady-state solution are used as the initial conditions for the time-dependent solution. The system under consideration is shown in Fig. 28. The equations that describe the surge tank dynamics must be included in the analysis, since the air-vapor mixture inside the surge tank plays an important role in generating and maintaining the oscillations. The continuity equation for the surge tank can be written as [138]:

$$\frac{d(P_s)}{dt} = P_{sa}^2 \frac{(G_1 - G_2)A_p}{P_{sao} V_{go} \rho_t} \tag{40}$$

where P_{sa} is the unsteady and P_{sao} is the steady state pressure of the air in the surge tank, V_{go} is the steady-state gas volume in the surge tank, and G_1 and G_2 are the inlet and outlet mass velocities to and from the surge tank, respectively. The momentum equation for the mass velocity from main tank to surge tank can be written as [138]:

$$G_1 = \left[\frac{(P_o - P_s)\rho_l}{R}\right]^{1/2} \tag{41}$$

where R is the restriction coefficient from the main tank to the surge tank, obtained from experimental data.

The steady-state solution, which is assumed to be valid at every point of the pressure-drop oscillation, is the same as that obtained in the previous section, which can be compactly written (between the surge tank and the system exit) as:

$$G_2 = G_2(P_s, Q_I) \tag{42}$$

During the oscillations, the heat input, Q_I, and surge tank pressure, P_s, change; thus, G_2 becomes the dependent variable of Q_I and P_s.

7.3.6. Comparison of steady-state results

Fig. 29 shows the pressure drop versus mass flow rate results for a constant inlet fluid temperature ($T_i = 20$ °C), with various electrical heat input rates ($Q_o = 0$–500 W). The theoretical predictions can be seen to be in very close agreement with the experimental results over the entire range of parameters involved. The region of negative slope

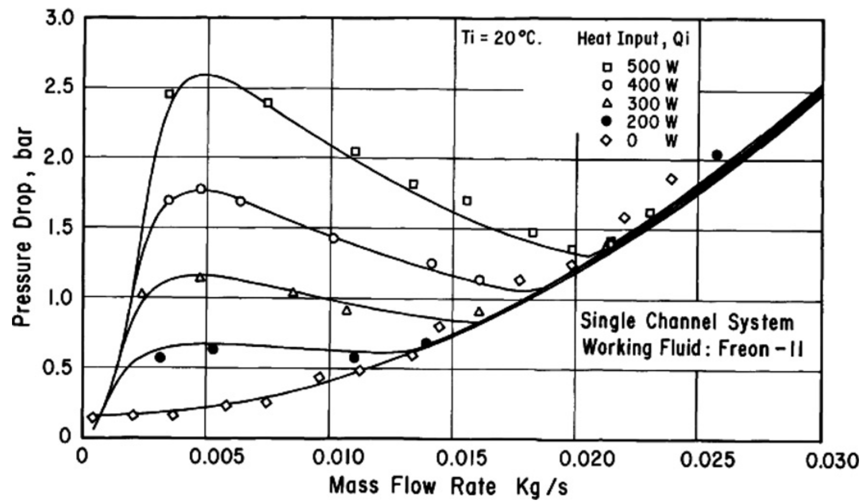

Fig. 29. Predicted steady-state pressure-drop characteristics at various heat inputs (Ti = 20 °C).

between the regions of positive slope plays a critical role in generating and sustaining oscillations.

Fig. 30 shows the effects of the degree of inlet subcooling, as defined by the difference between saturation temperature at surge tank pressure and actual inlet temperature, on the system characteristics. The pressure-drop versus mass flow rate curves are plotted for a constant electrical heat input rate ($Q_o = 500$ W), with various inlet temperatures ($T_i = (-8\ °C) - (+32\ °C)$). As the degree of inlet subcooling increases, the system becomes more unstable, as shown by the increasing magnitude of the negative slope.

7.3.7. Comparison of time-dependent solutions

For constant electrical heat input rate ($Q_o = 500$ W), with various inlet fluid temperatures and mass flow rates. The comparison of these results is summarized in Tables 5 and 6 [117]. Fig. 31 shows the theoretical prediction for the thermal oscillations of the wall temperature near the end of the heater.

In another work of Cao [141] and Cao et al. [142] the drift-flux model is adopted to predict the steady-state characteristics of the boiling system of the pressure-drop type oscillations in the upflow boiling system with an exit restriction. The effect of thermal non-equilibrium on the steady-state characteristics, oscillation periods at different heat inputs and inlet temperatures are also presented and verified by experimental findings given in [33].

$$\frac{\Delta P_{tp}}{\Delta P_{\mathrm{Lo}}} = 1 + \left(\frac{\upsilon_{\mathrm{v}}}{\upsilon_{\mathrm{l}}} - 1\right) c_1 x^{c_2} \tag{43}$$

In this study the following expression for the exit restriction has been developed theoretically and verified experimentally. Where c_1 and c_2 are correlation constants which are tabulated in Table 7.

7.3.8. Steady-state characteristics

Mathematical representation of the experimental system is shown in Fig. 29, where five distinct regions are identified

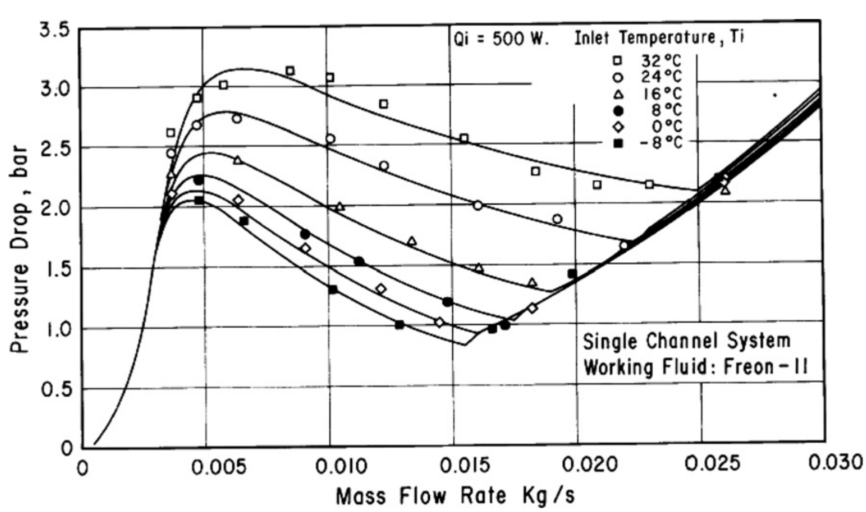

Fig. 30. Pressure-drop versus mass flow rate for different inlet temperatures. Comparison of steady-state model predictions with experimental data ($Q = 500$ W).

Table 5
Comparison of the experimental and theoretical results

Inlet temp. (T_i) (°C)	Mass flow rate (g/s)	Oscillation type	Location	Experimental		Theoretical	
				Period (s)	Amplitude (°C or bar)	Period (s)	Amplitude (°C or bar)
−8	5.42	Temperature	Heater wall (last T.C.)	115	92	115	95
		Pressure	Heater inlet	115	1.1	115	1.1
0	4.97	Temperature	Heater wall (last T.C.)	80	60	77	60
		Pressure	Heater inlet	80	1.02	77	1.05
8	6.82	Temperature	Heater wall (last T.C.)	75	75	73	75
		Pressure	Heater inlet	75	0.66	73	0.60
16	8.20	Temperature	Heater wall (last T.C.)	45	33	45	35
		Pressure	Heater inlet	45	0.54	45	0.50

Heater: I.D. 0.43″; O.D. 0.50″. Material: Stainless Steel. Heat input: 500 W.

Table 6
Comparison of the experimental and theoretical results

Inlet temp. (T_i) (°C)	Mass flow rate (g/s)	Oscillation type	Location	Experimental		Theoretical	
				Period (s)	Amplitude (°C or bar)	Period (s)	Amplitude (°C or bar)
−8	5.4	Temperature	Heater wall (last T.C.)	80	128	80	135
		Pressure	Heater inlet	80	0.24	80	0.45
0	6.8	Temperature	Heater wall (last T.C.)	50	85	50	75
		Pressure	Heater inlet	50	0.55	50	0.60
8	6.35	Temperature	Heater wall (last T.C.)	56	114	56	115
		Pressure	Heater inlet	56	1.26	56	1.35
16	9.04	Pressure	Heater inlet	30	48	30	45
		Pressure	Heater inlet	30	1.20	30	1.00

Heater: I.D. 0.305″; O.D. 0.375″. Material: Stainless Steel. Heat input: 500 W.

along the system, each having different characteristics. The components upstream of the inlet of the heated channel are lumped together and considered as the first region. The whole length of the heater is divided into the subcooled liquid region and the boiling two-phase fluid region, which are numbered the second and third regions. Region four extends from the exit of the heater to the exit restriction. The restriction exit is treated separately as region five.

7.3.9. Two-phase mixture region

Drift-flux model is used to formulate the problem in the two-phase mixture region. The time-smoothed, one-dimensional conservation equation as derived by Ishii and Zuber [80] are:

Conservation of mass of the mixture

$$\frac{\partial(\rho u)}{\partial z} = 0 \tag{44}$$

Conservation of mass of the vapor phase

$$\frac{\partial(\alpha \rho_v u v)}{\partial z} = \Gamma_v \tag{45}$$

Conservation of momentum of the mixture (neglecting the effect of surface tension)

$$\rho_m u_m \frac{\partial u_m}{\partial z} = -\frac{\partial P}{\partial z} - \frac{f_m}{2D_h}\rho_m u_m^2 + g\rho_m - \frac{\partial}{\partial z}\left(\frac{\rho_l - \rho_m}{\rho_m - \rho_v}\frac{\rho_l \rho_v}{\rho_m} V_{vj}^2\right) \tag{46}$$

Conservation of energy of the mixture (neglecting the effect of the kinetic and potential energy):

$$\rho_m u_m \frac{\partial h_m}{\partial z} = \frac{q_w \xi_h}{A_c} - \frac{\partial}{\partial z}\left(\frac{\rho_l - \rho_m}{\rho_l - \rho_v}\frac{\rho_l \rho_v}{\rho_m} V_{vj}(h_g - h_l)\right) \tag{47}$$

The mixture density, ρ_m, the mixture pressure, P_m, the mixture velocity, u_m, and the mixture enthalpy, h_m, are defined as:

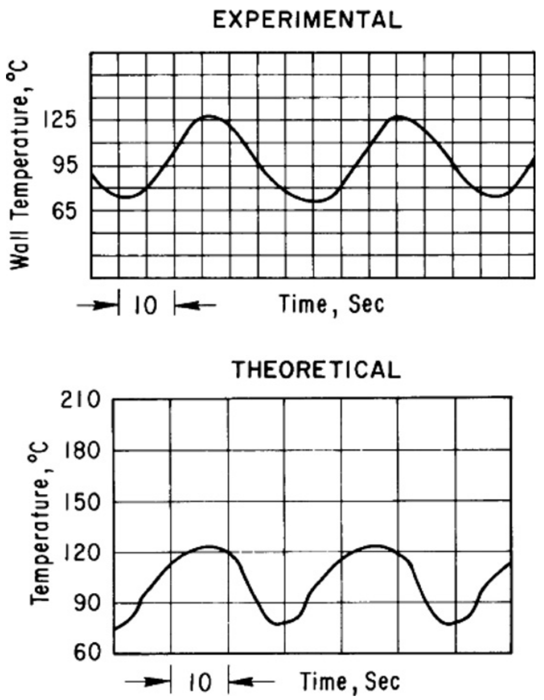

Fig. 31. Thermal oscillations at the wall ($Q = 500$ W, Ti = 16 °C, $\dot{m} = 9.04 \times 10^{-3}$ kg/s).

Table 7
Constants for c_1 and c_2 Eq. (43)

Heat input (W)	c_1	c_2
1000	2.4	1.5
800	2.3	1.5
600	1.5	1.5
400	1.0	1.5

$$\rho_m = \psi\rho_v + (1-\psi)\rho_l \tag{48}$$

$$P_m = \psi P_v + (1-\psi)P_l \tag{49}$$

$$u_m = \frac{\psi\rho_v u_v}{\rho_m} + \frac{(1-\psi)\rho_l u_l}{\rho_m} \tag{50}$$

$$h_m = \frac{\psi\rho_v h_v}{\rho_m} + \frac{(1-\psi)\rho_l h_l}{\rho_m} \tag{51}$$

Instead of solving the problem in a general form, the following simplifications assumption is introduced:

1. There is no pressure difference between the phases:

$$P_m = \psi P_v + (1-\psi)P_l = P = P_l = P_v \tag{52}$$

2. The enthalpy of the vapor phase is constant and is equal to the corresponding saturation value.
3. Liquid phase is incompressible.

7.3.10. The kinematic correlation for void fraction

The model used in the present study was proposed by Zuber and Findlay [128]. According to their analysis, the vapor velocity,, is related to the volumetric flux as:

$$u_v = C_o j + V_{vj} \tag{53}$$

where C_o is the distribution parameter, V_{vj}, is the drift velocity of the vapor phase with respect to the center of volume of the mixture and j is the volumetric flux defined as:

$$j = u_l(1-\psi) + u_v\psi \tag{54}$$

The following expressions are reported to give good results irrespective of the flow pattern [25].

$$C_0 = 1.13 \tag{55}$$

$$V_{vj} = 1.41\left[\frac{\sigma g(\rho_l - \rho_v)}{\rho_l^2}\right]^{1/4} \tag{56}$$

The comparison of the theoretical and experimental values of oscillation periods at different heat input is shown in Tables 8 and 9 for equilibrium and non-equilibrium models.

7.4. Horizontal system-drift-flux model

Fig. 32 is a schematic diagram of a multi-channel in tube boiling horizontal apparatus which is used in the experimental and theoretical study of two-phase flow instabilities of R-11. In their solutions one-dimensional time-dependent equations were used [143,144]. In the following, the mathematical formulation of the drift-flux model is presented.

Table 8
Comparison of experimental and theoretical results of pressure drop oscillations (bare tube; tube I.D. = 7.5 mm; exit restriction = 1.6 mm; inlet temperature = 23 °C)

Heat input (W)	Mass flow rate (g/s)	Theoretical period (s)	Experiment (s)
400	11.89	15.1	13.5
600	11.89	21.0	17.0
800	11.89	36.8	28.5
1000	11.89	51.5	32.5

Table 9
Comparison of experimental and theoretical results of pressure-drop oscillations (non-equilibrium model)

Temperature (°C)	Heat input (W)	Theoretical non-equilibrium period (s)	Theoretical (equilibrium) period (s)
10	400	34.8	37.6
	600	38.2	44.7
	800	37.3	45.2
	1000	35.8	44.6
0	400	57.5	65.0
	600	50.4	56.0
	800	48.4	54.8
	1000	43.1	52.4
−10	600	67.5	71.0
	800	58.4	66.9
	1000	51.8	60.7

Bare tube; tube id = 7.5 mm; exit restriction: 1.6 mm; $\dot{m} = 11.89$ g/s.

The study of two-phase flow dynamic instabilities, in general, requires the knowledge of the steady-state pressure-drop versus mass flow rate characteristics. The stability boundaries for pressure-drop and density-wave oscillations are usually shown on the plot of these relationships. These relationships, which are the steady-state solutions to the conservation equations, are also used to determine the initial conditions for both types of oscillations. Therefore, initially solutions are obtained for various heat inputs and/or inlet subcoolings under steady-state conditions.

7.4.1. Steady-state characteristics

Under steady-state conditions the two-phase flow governing equations from Zuber and Findlay [117] are:

Continuity equation:

$$\frac{\partial}{\partial z}[\rho_l u_l(1-\psi)+\rho_v u_v \psi]=0 \tag{57}$$

Energy equation:

$$\phi=\frac{\partial}{\partial z}[\rho_l u_l h_l(1-\psi)+\rho_v u_v h_v \psi] \tag{58}$$

Momentum equation:

$$-\frac{\partial P}{\partial z}=\frac{\partial}{\partial z}\left[G^2\left(\frac{(1-x)^2}{\rho_l(1-\psi)}+\frac{x^2}{\rho_v\psi}\right)\right]+\left(\frac{\partial P}{\partial z}\right)_{\mathrm{TP,fric}} + g[\rho_l(1-\psi)+\rho_v\psi] \tag{59}$$

The mass flux, G, in the momentum equation is defined as:

$$G=\rho_l u_l(1-\psi)+\rho_v u_v\psi \tag{60}$$

The quality (dryness fraction), x, is:

$$x=\frac{\rho_v u_v A_v}{\rho_v u_v A_v+\rho_l u_l A_l} \tag{61}$$

The void fraction, ψ, is:

$$\psi=\frac{A_v}{A} \tag{62}$$

The total enthalpy at a given section is:

$$h=\rho_l h_l(1-\psi)+\rho_v h_v\psi \tag{63}$$

7.4.2. Two-phase frictional pressure-drop

To estimate the magnitude of the two-phase frictional pressure gradient, a two-phase friction multiplier F_{M} is used along with an expression for the single-phase pressure-drop. F_{M} is defined as:

$$F_M=\frac{\Delta P_{\mathrm{TP}}}{\Delta P_{\mathrm{SP}}} \tag{64}$$

This factor is a function of the quality of the two-phase flow. The single-phase pressure gradient can be estimated by using the definition of the Fanning friction coefficient as:

$$\left(\frac{\mathrm{d}P}{\mathrm{d}z}\right)_{\mathrm{sp,fric}}=4\frac{f_o}{d}\frac{G^2}{2\rho_l} \tag{65}$$

where f_o is the single-phase friction factor estimated using Blasius' formula Eq. (19)

$$\left(\frac{\mathrm{d}P}{\mathrm{d}z}\right)_{\mathrm{TP,fric}}=4\frac{f_o}{d}\frac{G^2}{2\rho_l}F_M \tag{66}$$

7.4.3. The kinematic correlation for void fraction

To solve the three conservation equations presented above, relationships between the phase velocities in terms of the volumetric flow rates and void fraction need to be determined. The model to be used in the present study was originally proposed by Zuber and Findlay [128]. A volumetric flux for the two-phase mixture is defined as:

$$j=u_l(1-\psi)+u_v\psi \tag{67}$$

Equating the mass velocity of the liquid at any section to the liquid fraction of the total mass velocity, the following is observed:

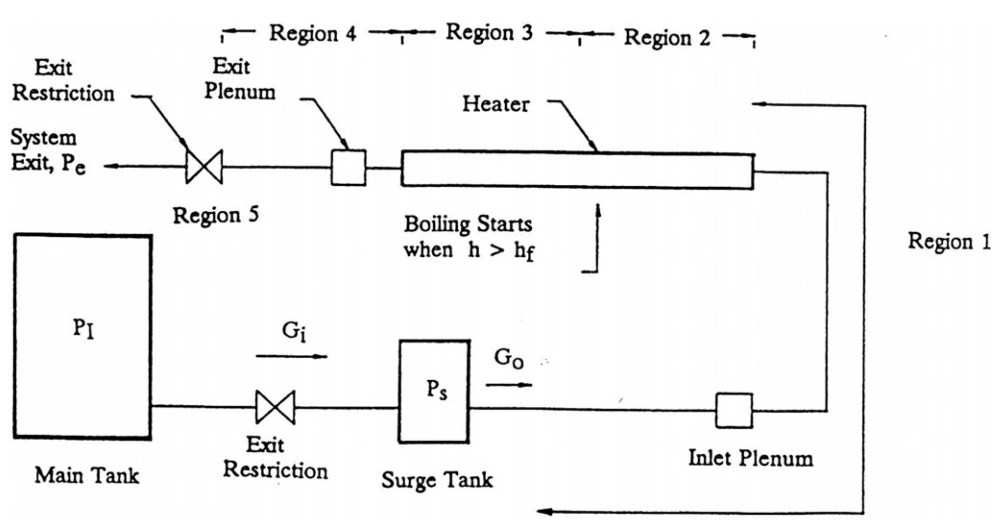

Fig. 32. Schematic diagram of the mathematical model for horizontal system.

$$\rho_l u_l (1-\psi) = G(1-x) \tag{68}$$

Similarly, it can be written for the vapor mass velocity:

$$\rho_v u_v \psi = Gx \tag{69}$$

Thus, the volumetric flux j can be expressed in terms of the mass velocity, G, as:

$$j = \frac{G(1-x)}{\rho_l} + \frac{xG}{\rho_v} \tag{70}$$

According to the Zuber-Findlay model [128], the vapor velocity, u_v, may be related to the volumetric flux as:

$$u_v = C_o j + u_{vj} \tag{71}$$

where C_o is the distribution parameter and u_{vj} is the drift velocity of the vapor phase with respect to the center of mass of the mixture. Where C_0 and u_{vj} are given by expressions (55) and (56).

In writing the finite-difference equations, five distinct regions are identified along the system shown in Fig. 5, each having different characteristics (Fig. 32). They are: the upstream tubing, the subcooled region of the heater, the boiling region of the heater, the region between the heater and the exit restriction, and the exit restriction. At the exit restriction, which is a sharp-edged orifice of diameter 2.64 mm, two-phase flow is observed. An empirical correlation, based on experimental data, is used to calculate the pressure-drop across the restriction. The two-phase pressure-drop across the restriction is found from:

$$\Delta P_e = \Delta P_{SP} F_M \tag{72}$$

where the single-phase pressure-drop across the orifice plate, ΔP_{SP}, was experimentally determined as:

$$\Delta P_{SP} = 175 \frac{G^2}{\rho_l} \tag{73}$$

The two-phase multiplier, F_M, has been experimentally determined:

$$F_M = 1 + 28.73 x_e - 6.68 x_e^2 + 25.518 x_e^3 \tag{74}$$

where x_e is the quality of the liquid vapor mixture at the exit of the heated section.

It is assumed that the wall temperature is constant across the thickness of the wall. The instantaneous heat transfer coefficient, α_i, under oscillatory conditions is determined using the following correlation [104]:

$$Nu = 190 C_s \left(\frac{P}{P_{crit}}\right)^{0.25} \left(\frac{qd}{\Delta h_v h_l}\right)^{0.7} e^{-0.125x} \tag{75}$$

where $Nu = \alpha_i d / k_l$ is the Nusselt number and $q = Q_I / \pi d L_h$ is the heat flux density.

7.4.4. Method of solution

Eqs. (38) and (39) are written in finite difference forms. In the pressure-drop oscillation model, fluid parameters and properties are calculated along the system during the oscillations. The fluid temperature inside the heater at any node is known. The heat transfer coefficient is calculated using Eq. (75) was calculated. The heat input into the fluid, Q_I, is assumed to start with. Then the heater inside wall temperature, T_{wi}, can be calculated using Eq. (38). During the oscillations, the heat transfer coefficient and the heat input change, and the heater wall temperature changes accordingly. The new heater wall temperature is now substituted in Eq. (39) to yield the heat input, Q_I, into the fluid. Using this value of the heat input the new heat transfer coefficient using Eq. (75) was calculated. This procedure is repeated to obtain the limit cycles. The solution of this procedure yields the thermal oscillations at any node of the heater. The rising portion of the pressure-drop oscillations corresponds to an increasing vapor flow that carries away more heat, thus lowering the wall temperature. The decreasing portion corresponds to a decreasing liquid mass flow rate that convects progressively lower heat away from the wall. This causes the temperature to increase. Tables 10 and 11 summarize the comparison between the experimental and theoretical results. The pressure-drop oscillations result through the interaction between the flow and the compressible volume in the surge-tank. Under the present experimental conditions, high-frequency density-wave oscillations also occur, which are superimposed on the pressure-drop oscillations. Tadrist [145] presented a brief review on two-phase flow instabilities in narrow spaces. The main instability types are discussed according to the existing experimental and theoretical results.

Table 10
Comparison of results from the drift-flux model and experimental studies

Mass flow rate (kg/s)	Heat input (W)	Pressure-drop (bar)	
		Experimental	Theoretical
0.02	0	0.08	0.1
0.02	1500	1.8	2
0.02	2000	2.8	3
0.02	2500	5.8	6
0.04	0	0.31	0.34
0.04	1500	2.0	2.2
0.04	2000	3.9	4.1
0.04	2500	7.0	7.2
0.06	0	0.73	0.75
0.06	1500	1.8	2

Steady-state characteristics; horizontal single channel; working fluid: R-11; tube I.D. = 8.34 mm, O.D. = 10.6 mm; exit restriction = 2.64 mm.

Table 11
Comparison of experimental and theoretical results; pressure-drop type oscillations in a horizontal single channel system; working fluid: R-11; tube I.D. = 8.34 mm, O.D. = 10.6 mm; mass flow rate = 0.0717 kg/s

Exit restriction (mm)	Heat input (W)	Period (s)	Amplitude (bar) experimental	Period (s)	Amplitude (bar) theoretical
2.64	2000	18	1.5	24	1.9
2.64	2500	16	2.2	18	2.0
3.175	2000	13	1.1	12	1.25
3.175	2500	14	1.2	12	1.4

8. Conclusions

Two-phase flow dynamic instabilities are time dependent complex phenomena encountered in two-phase flow heat exchanger equipment. All the experimental analysis have indicated the existence of the three major modes of oscillations, i.e. density-wave (high frequency), Pressure-drop (low frequency) and thermal oscillations in single and multi-channel, electrically heated, forced convection upflow and horizontal systems.

The physical nature of each mode of the oscillations is different and distinct that separate solution methods which are developed for each case. Well established experimental results are available in the literature for various two-phase flow systems under various conditions. The formulation of the conservation equations for general modeling of two-phase phenomenon is also available. This review indicates that in most of the theoretical predictions of amplitudes and periods of the sustained oscillations by the two models, namely homogenous flow and drift-flux models are validated by experimental findings. The stability boundaries for various operating conditions are obtained. In the most general formulation of the two-phase flow problem, the conservation equations are written separately for each of the phases which is called separated-flow model. Although the separated-flow model is the most satisfactory in theory, it is complicated to use in problems of practical importance because of the seven constitutive laws that are required. Therefore separated-flow modeling is open for further research. Very valuable experimental results are available to validate the models used. All the experimental and theoretical studies are made extensively for tube boiling in macrochannels. Two-phase flow studies in microchannels in general and two-phase instabilities in particular are becoming very important. Microchannels are used in microdevices having the dimensions in the range of μm in many fields such as biomedical applications, space industry and microelectromechanical systems (MEMS) which are an open area for researchers to study two-phase flow instabilities in microtube boiling systems.

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